Mathematics Education Around the World:
Bridging Policy and Practice

Reflections from the 2001 Park City Mathematics Institute International Panel on Policy and Practice in Mathematics Education

19-23 July 2001

Institute for Advanced Study

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This report is the product of an international seminar on mathematics education held at the Park City Mathematics Institute on 19-23 July 2001 at Park City, Utah under the auspices of the

Institute for Advanced Study
Princeton, NJ

The report is available online at http://mathforum.org.pcmi/

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Acknowledgements

The International Panel could not have been the success that it was without the extraordinary efforts of a number of individuals.

We would like to acknowledge the staff from the Division of Science and Mathematics Education at Michigan State University and Park City Mathematics Institute for their efforts in putting the panel and this manuscript together. In particular, Jean Beland and Nancy DeMello were instrumental in overseeing the arrangements for the meeting and for providing onsite support during the meeting. We thank Catherine Giesbrecht who served as our liaison to the Institute for Advanced Study, managed the contacts with the international participants, and facilitated our interaction with the Park City Mathematics Institute. We are also grateful to Carol Hattan for arranging to have members of the High School Teachers’ Program serve as hosts for the participants. And, we thank Herb Clemens for conceptualizing the panel and for his vision, persistence, and support as we worked through the details of the project. We also thank Elaine Wolfensohn for her encouragement and advice as the panel became a reality.

We are grateful to our funders, the Wolfensohn Family Foundation, the Bristol-Myers Squibb Foundation, and International Commission on Mathematics Instruction, for believing that the idea of an international panel in mathematics education had merit and for providing us with the resources to make it a reality.

Joan Ferrini-Mundy
Gail Burrill
Project Directors
November, 2002
Table of Contents

Acknowledgements     page 3
Introduction          page 5
Issue 1: National Curriculum, Standards, and Teaching Practice  page 10
Issue 2: Teacher Education Process and Teacher Practice     page 26
Issue 3: Content in the Mathematics Curriculum           page 35
Issue 4: Tradition and Reform in Mathematics Education   page 47
Issue 5: Depth and Breadth in the Mathematics Curriculum   page 56
Issue 6: Excellence and Access                           page 68
Issue 7: Math Education as a Profession                  page 91
Appendix A   Agenda                                      page 99
Appendix B   Participants                                 page 102
Appendix C   Background Materials                        page 104
Appendix D   Regulation of the French Educational System for Mathematics  page 105
Appendix E   Four Case Studies from India                 page 112
Appendix F   Balance of Power and Inputs in the Swedish System  page 117
Introduction

Teaching children mathematics is a central element in educational systems across nations and within countries. International studies such as the Third International Mathematics and Science Study measure student achievement and collect information on factors about schools, teachers, and curriculum that may affect this achievement. Each country has its own struggles within the context of its own culture. An International Panel on Policy and Practice in Mathematics Education sponsored by the Institute for Advanced Study/Park City Mathematics Institute (PCMI) engaged in a stimulating five-day discussion about common issues and concerns in the teaching and learning of mathematics. The Wolfenson Family Foundation, the Bristol-Myers Squibb Foundation, and the International Commission on Mathematics Instruction funded the seminar. Teams of two educators—a university mathematics educator or policy-maker and a secondary teacher—from eight nations met to discuss major issues in mathematics education policy and practice. The seminar goals were to:

- promote open discussion of issues affecting the mathematics education policies and practices of each nation,
- identify common issues faced across national contexts,
- identify common sources of direction and support for efforts to address problems, and
- search for common solutions to related problems
The seminar, led by Joan Ferrini-Mundy, Michigan State University, and Gail Burrill, National Research Council, was organized to stimulate conversation and productive exchange of information that could serve as a basis for continued efforts to address issues in mathematics education.

Background

The eight nations represented in the seminar were Brazil, Egypt, France, India, Japan, Kenya, Sweden, and the United States. (See Appendix B for a list of participants.) The nature and impact of each nation’s policies and practices were filtered through the experiences of the individual members of the two-person teams. This document is meant to be a “story” that describes an international conversation about issues in mathematics education. The team members whose views are expressed in this report were not functioning in any way as official representatives of their nations of origin. Thus, the views expressed by the members of these teams and the information contained in this report are not intended to reflect the status of mathematics education in each nation. Although there is some discussion of the national mathematics education context, each individual brought a unique perspective to the discussions. As such, issues of region, locality or other circumstances may have influenced individual views and opinions. It is not the intention of the PCMI, or this report, to claim that the views expressed are indicative of the national situation in each country.

The primary goal of the seminar was to consider issues and challenges that are similar for mathematics educators from very different contexts and to look across countries for common strategies and approaches. Researchers Deborah Loewenburg Ball and Hyman Bass from the University of Michigan in the U.S. began the seminar with a video of a third-grade class with
children of many languages and cultures learning about odd and even numbers. Viewers were asked to consider what students might be learning, how students deal with mathematical ideas, and what a teacher might need to know to handle what was taking place.

During the discussion following the video, participants observed that the students seemed to have learned certain norms of behavior relating to mathematics and mathematical reasoning, such as the need to argue from a definition and their use of mathematical language. They noted that the teacher did not impose her view but allowed the situation to unfold and raised a question about how far she would attempt to take their understanding. Ball and Bass encouraged the participants to think about the video as a way to stay grounded in practice as the seminar discussion continued and to use the way the children were learning together as a model for the work of the seminar, asking for clarifications and definitions, creating norms for agreement and disagreement, and building on ideas and developing them further.

The seminar discussion was framed by seven questions, identified in advance by the organizers, covering a broad spectrum of issues related to both policy and practice in mathematics education.

1. What is the relationship of national standards and national curriculum to teaching practice in classrooms in your country?
2. What is the system of teacher education in your country and how does it relate to teaching practice?
3. Case study inquiries: Describe the role of algebra in the middle and secondary mathematics curriculum in your country. How are ideas from probability and statistics currently configured in your system?
4. How does your country handle the balance between tradition and reform in mathematics education? What do tradition and reform mean within your mathematics education system?
5. How does your educational system decide the balance between depth and breadth (i.e. between insistence on in-depth knowledge of relatively fewer core topics vs. a broad inclusion of topics, with less emphasis on each)? How is this decision effected in practice?
6. How does your country and culture deal with the challenges of excellence and accessibility in mathematics education? What is the balance of poser and input into the system between the various educational constituencies?

7. What is the role of mathematics education as a profession and of mathematics education research in your country?

A team from one of the countries introduced each question (Table 1), followed by group discussion highlighting the similarities and differences noticed by those from other nations.

<table>
<thead>
<tr>
<th>Table 1. Focus Questions</th>
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<tr>
<td><strong>Issue</strong></td>
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<tr>
<td>1. National Curriculum, Standards, &amp; Teaching Practice</td>
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<td>2. Teacher Education Process &amp; Teaching Practice</td>
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<tr>
<td>3. Algebra, Statistics &amp; Probability in Math Curriculum</td>
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<td>4. Tradition &amp; Reform in Math Education</td>
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<td>5. Depth &amp; Breadth in the Mathematics Curriculum</td>
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<td>6. Excellence &amp; Access in Math Education</td>
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<td>7. Role of Math Education &amp; Math Education Research</td>
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</table>

Following the discussion of each topic, two observers, Hyman Bass from the United States and Hiroshi Fujita, Tokai University from Japan, reflected on the discussion from their individual perspectives. Throughout the forum, participants were able to revisit topics after the initial discussion by posting their thoughts, ideas, and questions on bulletin boards dedicated to each question. (See Appendix A for the agenda, Appendix B for information about the participants, and Appendix C for a list of background materials provided for the participants.)
Discussion Summaries

The summaries that comprise the remainder of this document contain the opening statements presented by the members of the team to whom the question was assigned, a table presenting the similarities and differences noted by the participants from each nation, a thematically arranged interpretation of the discussion content, and key questions that arose in the discussion. These summaries are intended to 1) convey common assumptions, common language, and shared definitions that arose from the discussions, 2) capture “insights” expressed by the participants, 3) convey emerging themes that arose in the discussion of the issues, and 4) provide a basis for continuing the conversations.
Issue 1: National Curriculum, Standards, and Teaching Practice

What is the relationship of national standards and national curriculum to teaching practice in classrooms in your country?

France: Antoine Bodin and Catherine Sackur

Opening Statements

Antoine Bodin
Université de France Compte IREM

The main question I am supposed to answer in this short presentation is: How does your education system function, both in theory and practice? And more precisely: What is the relationship of national standards and national curriculum to teaching practice in classrooms in France? I will try to answer part of these questions from a research and curriculum development point of view. Later, when Catherine speaks, she will tackle these questions more from a teacher’s perspective.

Concerning the question regarding the relationship between national curriculum/standards and teaching practice, an answer that immediately comes to mind is that in France, indeed, this relationship should be both direct and obvious. France is a centralized country, so standards and curriculum are decided at the national level and apply for all in the country. It follows that the whole process should be a hierarchical and compulsory one. The reality, however, is much more complex.

An influential network of university institutes for mathematics teaching (e.g. the 26 IREM1 institutes across the country), along with the Professional Association of Mathematics Teachers,

1 Institutes de Reserches en Education Mathematique
makes it difficult for the Ministry to fully control teaching practice. Practice is actually influenced by both the centralized, hierarchical system, as well as by strong teacher individualism associated with a host of non-official influences\(^2\). Surprising though it may seem, the impetus for some of the non-official influences comes from the ministry itself.

In an effort to maintain a balanced and unified education system, France actually tries to keep the best of its centralization heritage while delegating some national responsibilities to the Regional Education Boards (30 Academies in France). For example, the Ministry has assigned the task of writing the official curriculum to a group known as the National Mathematics Expert Group\(^3\). While the ministry is primarily responsible for the development of the national curriculum, its development is best viewed as a recursive process involving teachers and policy-makers rather than as a process of delivering regulations.

To encourage good teaching practice and teachers’ personal involvement in the process, the ministry relies on several features and devices. These include:

- The National Curriculum designed in the Ministry
- National Examinations at grades 9 and 12 (especially the Baccalaureate)
- Diagnostic National Assessments at the primary level (grade 3), lower secondary level (grade 6), and upper secondary level (grade 10), and
- Incentives for teaching innovation.

The Power Point slides: Regulation of the French Educational System for Mathematics found in Appendix D present a system analysis of the French mathematics curriculum design and development process.

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\(^{2}\) Many people and bodies think that they have the authority and competence to speak when mathematics education is at stake. That feeling seems to be more prevalent towards mathematics than for any other subject.

\(^{3}\) This group is composed of university professors, mathematics inspectors, and secondary teachers (i.e. the 12 members of the GEPS).
To conclude my presentation and open discussion and comparisons with what is at issue in other countries, I offer several questions for consideration. These include:

- How can we promote a student-centered, activity-based, constructivist teaching approach without negatively impacting students’ proficiency?
- How can we set national standards without reducing teaching and learning activities to a collection of insignificant basic and procedural skills?
- How can we maintain national unity in the mathematics curriculum while allowing for openness, teacher initiative, and innovative practices?
- How can we preserve the value and reliability of the baccalaureate without interfering with curricular and teaching reforms?
- How can we take advantage of the links between mathematics and other topics, and facilitate integrative teaching without sacrificing mathematical rigor?
- How do we determine what mathematics content is more relevant for general education (i.e. mathematical literacy) and what is more relevant for those preparing to study advanced mathematics?
- Conversely, how do we identify which present mathematics topics can be safely removed from the curriculum?
- How can we maintain high standards without discouraging students from learning mathematics? And how do we choose which standards to uphold?

Catherine Sackur
IREM de NICE-UNSA

I am a high school math teacher. I teach students who are in the last three years of secondary school (age 15-18). Recently I’ve been teaching the very last year for students who have chosen an advanced course in mathematics. I am a researcher in mathematics education, and I participate in teachers’ education by supervising student teachers in their last year of training.4

I have been asked to speak on the relationship between national standards/curriculum and teachers’ practice in the classroom. First of all, I feel the need to clarify what we mean by

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4 During the last year of teacher training, student teachers teach one class.
national standards and national curriculum in our country. In France, a curriculum is a text written by experts chosen by the Education Department. A curriculum has three parts:

1) motivation for choices
2) a rather detailed list of the notions in the different chapters, and the expected competencies
3) comments on these competencies (e.g. “Students should know…” “It is possible to have students work on…but no specific knowledge on this point is expected…”). As the final examination is the same for all the schools, these comments are important. The basic knowledge tested by the examination is supposed to be the same everywhere.

I would like to approach the problem of relationship between national curriculum and practice in class from two different angles. In an effort to be a little concrete, I’ll begin with an example on the use of pocket calculators.

Around 1984, graphic calculators became less expensive and more accessible to students. Since then, the education department has emphasized the use of pocket calculators in elementary and secondary schools. There have been many efforts to integrate them into math education, and they became an active part of the math teaching at those levels. Teachers have been required to integrate pocket graphical calculators at all levels from primary school to the end of high school. In grade 10, students are expected to know how to trace the graph of a function. In grade 12, they use some of the software and programming features to study sequences, or some algorithms in number theory.

On the contrary, at the university level there has been no pressure to introduce pocket graphical calculators, and they are not used at all. So, we observe two things with regard to policy and practice:

1. the demands of the institution are really taken into account, but
2. the possibility exists that the policies conflict with preferred practice. For example, many teachers believe that number theory is a place for logical reasoning, not for algorithms and software.
Two questions may be used to organize the discussion. To what account do the demands of the curriculum influence practice in classrooms? And to what extent does classroom practice vary in our very centralized system? With regard to the first question, one can distinguish three trends in content and teaching:

- **Long lasting practice**
  - Use of calculators
  - Use of software

- **Medium** (These are no longer relevant. Many teachers still pretend to rely on these practices, but they have been totally changed in the long run. Sort of ritual.)
  - Pre-lesson activities
  - Modules
  - Experimentation in mathematics

- **Short-lived practice**
  - Scientific discussion
  - Open-ended problem-solving

To understand these phenomena, consider:

- **Innovative practice in the classroom places new demands on teachers.**
  - It is difficult to anticipate how the situation will unfold. Teachers must be prepared to improvise depending on students’ reactions.

- **The reactions of students and the society are very influential.**
  - Students are very reluctant to accept new ways of teaching. Also, there is an idea in society of what teaching mathematics should be, and it is very difficult to go against that notion.
  - Old versus new dialectic: New practice must not be too far away from old practice.

- **Problem of preservice and inservice training.**
  - At a high level, people who decide about the curriculum have a constructivist model of learning arising from the influence of researchers on math education.
  - At the university, the model of teaching generally is that if you are a good mathematician, you will teach well. You learn a theory, you apply it, and that’s all. The teachers have not learned to behave in a different way.

Although we have a very centralized system, there is still variability in classroom practice.

While the contents and timetable are the same for all schools, teachers are supposed to have a certain level of autonomy on different matters. So, how do teachers take into account the
expectations of the curriculum and education officers who visit them, and how do they use their autonomy in the classroom

**Variation within an individual teacher’s practice**

Teaching practice varies on two levels: within and between. Although national standards are provided, each teacher is more or less a creator of curriculum. Teachers vary their practice from year to year. They may choose to present the chapters in a different order or vary the weight given to different chapters. They may also choose to vary their approach to teaching the content based on the characteristics of students in the class.

I have noticed this in my own practice. I may read about something in a review for teachers and decided to experiment with it in my classroom. Sometimes my choice depends on the other teachers in the school. We may decide to collaborate and agree to emphasize a topic on which I do not usually spend more than the minimum expected time. Sometimes I choose to emphasize a particular notion if I think it is more useful than another in helping my students overcome their difficulties or if they seem particularly interested in a concept. For example, my decision to emphasize group structure is partly dependent on the students I have in my classroom. We have many opportunities to study group structure in geometry\(^5\), number theory\(^6\), and in the theory of complex numbers\(^7\). This year I’ve decided to emphasize this point. In another year, I might choose to spend more time on space geometry or on integrals and approximate calculation of integrals (e.g. the rectangle method).

**Variation between teachers**

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\(^5\) The group of isometries and some of its sub-groups, and in rotations/translations.

\(^6\) The different groups of congruences.

\(^7\) The group of complex roots of 1.
Teachers’ beliefs both about the nature of mathematics learning and the purpose of certain content influence their practice. Teachers are often reluctant to change their beliefs about these. For example, in the new curriculum, number theory is seen as an opportunity to introduce algorithmic thinking and the use of software, but as I mentioned earlier many teachers view number theory as a place where students can learn logical thinking (e.g. implication and logical equivalence).

Earlier I mentioned teachers’ preservice and inservice training. This issue of beliefs is one where teacher training is fundamental. Aside from the official preservice and inservice training, we have many opportunities to meet other teachers who develop rather different ways of teaching and who experiment with new practices in their classroom (e.g. “math en jeans”, mathematical rally, and narratives on some research activities such as open-ended problem solving). Some of these experiments have also influenced the experts who decide on the curriculum.

Considering these issues, we are led to ask two important questions about the relationship between policy and practice in classrooms: 1) how does a centralized system benefit students? and 2) how does variability in practice benefit students? Some additional issues we may want to consider during the discussion include:

- Increased access to school and the influence of this on practice and content
- The role that university teachers and society play the in determining the content of the curriculum
- Recent emphasis on practice over content in the curriculum
- Adapting math teaching to the evolution of mathematics and to the evolution of students.

Participants identified the similarities and differences represented in Table 2.
<table>
<thead>
<tr>
<th>Country</th>
<th>Similarities to France</th>
<th>Differences from France</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>National curriculum</td>
<td>The curriculum is actually totally controlled from within the classrooms.</td>
</tr>
<tr>
<td></td>
<td>Teacher freedom</td>
<td>The Brazil Mathematical Society has been influential but more through its members than as an institution. Many are commissioned in the ministry (e.g. teacher education program, collection of books for high school teachers). Research has little impact on math education. Unlike in France or the NCTM, researchers serve only as consultants to the ministry, not as a community or an organization. There are no exams related to getting a degree. It is decided in the classrooms whether or not students get a degree.</td>
</tr>
<tr>
<td>Egypt</td>
<td>National curriculum, strong central system.</td>
<td>Neither strong teacher organization nor associated with official influence University institutions and math society not important in process of curriculum development.</td>
</tr>
<tr>
<td>India</td>
<td>Central Curriculum</td>
<td>Education is the responsibility of the states. States are not compelled to follow the central curriculum. Most prepare their own, taking the major portion from the central curriculum. Calculators are not available to many students.</td>
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<td></td>
<td>Teacher freedom</td>
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</table>

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8 Curricula are seldom related to the difficulty involved in teaching one concept or another, rather they depend on some choice of university teachers or ideas that suit the society.

9 Analysis of data, number theory, algorithms, and the use of software.
<table>
<thead>
<tr>
<th><strong>Country</strong></th>
<th><strong>System</strong></th>
<th><strong>Curriculum</strong></th>
<th><strong>Examinations</strong></th>
<th><strong>Calculator Policy</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>Centralized</td>
<td>National Curriculum</td>
<td>In secondary school most teachers do not use calculators.</td>
<td>Calculators are not allowed in class at any level or during exams. Students must still use logarithm tables during exams. If the Examination Council finds students using calculators, their exam results are canceled.</td>
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<tr>
<td></td>
<td></td>
<td>A great deal of freedom for teachers and great deal of variability in practice</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A lot of innovation, a lot of interest in teaching methods, and curriculum (intended, implemented, and attained).</td>
<td></td>
<td></td>
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<tr>
<td>Kenya</td>
<td>Very centralized</td>
<td>National curriculum</td>
<td></td>
<td>Calculators are not allowed in class at any level or during exams. Students must still use logarithm tables during exams. If the Examination Council finds students using calculators, their exam results are canceled.</td>
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<tr>
<td></td>
<td></td>
<td>Compulsory National Exams at the end of primary and secondary school.</td>
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<tr>
<td>Sweden</td>
<td>School is compulsory for the first 9 years.</td>
<td>National Test (not examination) in grade 9.</td>
<td>The national curriculum in Sweden contains general goals, but does not specify what exactly the content should be, the order it should be learned, or in what specific year you should do a specific topic.</td>
<td>There is great freedom within the frames and goals to attain at years 5 and 9. Minimum competency goals describe what everyone should attain, and higher goals describe what all should work towards -reach as high as they can.</td>
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<tr>
<td></td>
<td></td>
<td>Freedom for teachers.</td>
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<td></td>
<td></td>
<td>Teachers can choose texts at the local level, and the texts are produced by various authors.</td>
<td></td>
<td>There are no official textbooks, and</td>
</tr>
<tr>
<td>Country</td>
<td>Policy and Practice</td>
<td>Mathematics Education</td>
<td>Implementation</td>
<td>Assessment</td>
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<tr>
<td>United States</td>
<td>Similar gap between preservice/inservice training, and what teachers actually do.</td>
<td>Not centralized. Real control of the schools is in the hands of local school boards and local community.</td>
<td>No national curriculum but in some states the curriculum is state controlled</td>
<td>No national exams, but there is a move in the current government to institute national testing through 8th grade.</td>
</tr>
<tr>
<td></td>
<td>In the U.S., everyone also feels they have the right and responsibility to talk about how math ought to be taught. Even if they say, “I was never good at math” they still will tell you how to teach it.</td>
<td>Standards were originally developed by teacher-led associations.</td>
<td>Federal, state, and local districts influence how things are done through funding requirements and state testing.</td>
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</table>

**Observer Commentary**

*Hyman Bass*

*University of Michigan*

Your comments raised many questions in my mind. I speak mainly from my experience here in the U.S. We are struggling with the same problems, but we have a system that is quite different. Except for the U.S., most of the countries have a national curriculum that is formed by somebody in a Ministry of Education. This is very different from the U.S. where there is a long tradition of local control. National forces make practice somewhat similar anyway. For example, commercial textbooks and tests make practice rather centralized. But this is not organized by a ministry of education. Even though you have a centralized curriculum, you have access to many
textbooks. This is surprising for Americans. We would like to know how variable these texts are, and how are they determined to be consistent with the national curriculum. Does anyone evaluate available texts? Do individual teachers make independent decisions?

There was a very interesting contrast in the use of calculators. Some used no calculators, and others used them widely. France had relatively open use, but calculator use is much less common in Japan. They are even available to students in India. Why is this? It would be interesting to learn more about this.

It was interesting to hear how much freedom teachers have even in systems that are centralized. This would be good for U.S. teachers to learn, because they resist centralized curriculum, thinking it means that they would not have professional authority. Even in your systems, teachers have much autonomy. This is an area that would be interesting to explore. What is the nature of teachers’ creative work even in nationalized systems?

I have questions about how these centralized curricula and standards are produced and how they are received. Who writes them, and what are their professional credentials? This is a special issue in the U.S. because we have no system for this. For example, mathematics standards were produced by the National Council of Teachers of Mathematics (NCTM), the professional organization of teachers, while the science standards were produced at the National Academy of Sciences. Who should do this work? We have a lot to learn, but we have no system for this. Also, how are these documents received and used by practitioners (people who work in schools with students)? How much do these documents guide the practice of teachers? How do they influence or constrain teachers’ autonomy? Do teachers see these documents as “friendly”
to their work or do they see these as coming from a community distant from their work, seeking to control their practice?

If we wanted to create such a system in this country, from what source we would take the people? What would qualify them to do this?

Hiroshi Fujita
The Research Institute of Educational Development, Tokai University

I was interested by the presentation of the French people and also by your discussion. At this moment, I want to present my impression of the formulation of the problem. A national curriculum and the educational system can be a means of controlling education. The implementation of a national curriculum must be done by a teacher with appropriate freedom. National curriculum set by the government and local initiatives set mostly by teachers modify these national curricula. Local educational authorities also play a role in this modification.

At the Ninth International Congress on Mathematics Education (ICME-9), a speaker from the ministry in Singapore explained that in Singapore there is a national curriculum instituted by the government. Still, a great deal of freedom is given to the schools. In Japan, local initiatives are maintained mostly by teachers. Schools and local committees work for better implementation of the national curriculum.

For a better national curriculum, there needs to be a feedback process. What I heard from France and what I see in Japan is that the government or ministry sets a national curriculum, and they set up councils or committees to help implement the curriculum. Money is spent to disseminate the curriculum and train the teachers for the change. Teachers give input to the
ministry about the curriculum, and organizations such as universities, educational institutes, and projects become involved in making these adaptations.

In Japan, the Academic Society for Mathematics, Applied Mathematics, and Science Education give input. Our ministry is not very tied to this Society at this moment, which is asked to cooperate and advise the councils and committees. The government is very strong over schools, but they are very weak over mass communication. Industry seeks a strong influence over the ministry. There is a tendency to underestimate the role and value of mathematics and science.

Hashimoto said that calculators are not frequently used in Japanese classrooms, but actually the national curriculum encourages the use of calculators. However, the entrance exams to university do not permit calculators, so this has an influence. The value of calculators in education differs according to the level of pupils. In elementary school, we should be very careful. For upper elementary we can use them carefully. In the lower secondary school, calculators can be useful and should be encouraged. The value of calculators cannot be discussed without description of how they will be used. I support their use because they can help students develop in pupils an appreciation of mathematics.

Themes That Emerged From the Discussion

Theme 1: Centralization and autonomy

Educational systems in different countries vary in terms of how centralized and regulated they are and in terms of how centralization is achieved. A variety of mechanisms are employed in an attempt to control what happens in mathematics classrooms. These include the
establishment of national curricula and national (high-stakes) examinations, the choice of approved textbooks, and regulations regarding acceptable instructional practices, such as the use (or not) of calculators in the mathematics classroom.

**Illustrative quotes**

“There is a general tendency for decentralized systems of education to attempt some centralized procedures (e.g. national aims, curriculum, standards committees, national tests and vice versa). Centralized systems may attempt to extend authorities to local units.” (Mina)

“One of the factors [in the success of centralization] is the degree of freedom that the teachers enjoy in teaching in a country. The issue of centralization might not be as such the major issue responsible for the outcomes of mathematics education in a particular country.” (Mina)

[One of the important issues involves] how to keep in focus both the small scale local situation on an everyday basis, and also to look at the global situation and consider how things will work on the large scale. (paraphrase of Lins.)

**Theme 2: Standards, goals and mathematical content**

Another central theme that arose from the discussion was related to the scope of national standards and curricula. On the one hand, standards that are too narrow or prescriptive can result in a mathematics curriculum that is less challenging than what was intended. On the other hand, standards that are too broad or "loose" can be ineffective, in that they enable any type of mathematics instruction to fit under the "umbrella" of standards. There is concern about this in the United States.

**Illustrative quotes**

“In India, there is an effort to work out minimal levels of learning (MLL). The idea is that at a certain stage the child will acquire these skills by certain grades (lists of skills at end of grades). The MLL influences curriculum framing and classroom teaching. The negative aspect is that many teachers consider this as a maximum level of learning and don’t go beyond.” (Agarkar)
National guidelines can only set content recommendations. There are many arguments because people try to use broad goals such as “thinking algebraically” as content. Feedback says that standards such as this are difficult for others to understand. (Paraphrase of Lins)

**Key Questions**

Key questions about national standards and curriculum include those related to:

*The structure of the system*

- What does it mean for an educational system to be centralized (or not)? Are we working from a shared definition? In the discussion we saw great variation across our countries.
- How does centralization influence what teachers do?
- How is centralization of an educational system achieved? And who are the players?
- How might research influence policies aimed at centralizing mathematics teaching and learning?
- What are the implications of centralization policies for classroom practice? For example, what are the implications of approving only certain mathematics textbooks on instruction? On student achievement? How much autonomy do teachers actually have in a centralized educational system, and how is it exercised?
- Given that most mathematics teachers have considerable autonomy to make their own decisions regarding what happens in their classroom, even in settings with very strong regulations and mandates of their country/state/local agency, what are the implications for the development and nature of policies aimed at centralizing mathematics teaching and learning? Some centralization policies are conceptualized with the assumption that teachers maintain autonomy; what are their characteristics? What roles should mathematics teachers play in the establishment of policies aimed at centralizing mathematics instruction (i.e. development of a national curriculum and national exam)?
- What is the underlying view of mathematics teachers and their competence/efficacy in highly centralized and in decentralized educational systems? What assumptions about teaching and learning are being made?
- To what extent is mathematics instruction influenced by mandated examinations?

*The nature of standards*

- What are the implications of content goals that are so highly specified that they are measurable?
- Can standards allow for focus on "knowledge with understanding" and also be specific enough to define assessment expectations?
- What is the dynamic of moving to high standards or to more inclusive standards in terms of student access and learning?
The nature and purpose of math instruction

- What mathematics should students learn? Why should students learn certain mathematical content? Education through mathematics and education for mathematics: what are the implications of each?
- How much freedom is there for content choices?
- What is the role of technology in learning mathematics?
- What are the implications of massification\(^\text{10}\) of mathematics education?

\(^{10}\) Massification: increased access for the general student population.
Issue 2: Teacher Education Process and Teacher Practice

What is the system of teacher education in your country and how does it relate to teaching practice?

Egypt: Fayez Mina and Khaled Farouk Etman

Opening Statements

Fayez Mina
Aim Shams University

In Egypt, examinations are a driving force in shaping what teachers do. The curriculum is traditional in almost all respects. Mathematics is taught (approximately) entirely separate from other disciplines and life activities. Little attention is given to educational activities, employing educational media, or advanced technology.

Few teachers at the kindergarten level are university graduates, but the majority have graduated from secondary school. At the primary level the majority of teachers are intermediate graduates from an education program, with or without specialization in mathematics. A 1989 policy mandated that all teachers of primary education should have a university degree, specializing in education. The majority of secondary teachers are university graduates with an education background, and almost all specialize in mathematics.

There are some calls that the teacher preparation program should consider the process of professionalization of teacher education, including work with daily school activities and practicing ideas of recommended teaching (e.g. participation with students). These are not common in faculties of education in Egypt. Inservice teacher education is traditional and routine with little coherence or organization. There are no common scenarios for what will or should happen in most inservice programs. A video conference system with the maximum possibility of
having one training session (approximately one week of half-day sessions) for each teacher every four years is in place. However, even with this system it is difficult to reach all of the teachers, and there is no attempt to follow-up after training.

There is a connection between level of teaching and social status – teaching at the upper secondary level has the highest status. Tension exists between public education and private lessons. The official income of teachers is very low, while a high income can be achieved by giving private lessons. (One day of private lessons can pay several times more than a teacher’s official salary). This contradiction raises many questions about teaching as a profession in the country. The only specialized association in the country is the newly established Egyptian Society for Mathematics Education.

Khaled Farouk Etman
Orman Experimental Secondary School

The teachers’ aim in teaching mathematics is to meet whatever is in the syllabus and help prepare their students for exams. To achieve these goals, teachers focus their daily activities on topics related to the exam. To help students earn high scores on the exam, they rely on tutoring and also rewrite content in textbooks to provide more examples and famous exercises that consistently appear on examinations. There is a very large communication gap among teachers in classrooms, between teachers and their inspectors, and between teachers and policy-makers. However, teachers are aware of the expectations for their performance. According to the evaluation criteria, an expert teacher should be flexible in content knowledge, attentive to students’ ability, and knowledgeable about problem solving techniques.
In order to achieve these goals and help their students achieve high scores, teachers have to
a) understand the syllabus, how it was developed and its aim, b) know what mathematics
knowledge is necessary for a generation that shares values of its society, and c) know the
character of schoolbooks and teachers’ guide books.

Participants identified the similarities and differences represented in Table 3.

**Table 3 Egypt: Similarities and Differences Among Participating Countries with Respect to
Teacher Education**

<table>
<thead>
<tr>
<th>Country</th>
<th>Similarities to Egypt</th>
<th>Differences from Egypt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Teachers do not talk to ministry; Teaching is much work, little pay, and not highly respected as a profession. In practice use textbooks, not too flexible.</td>
<td>Teacher education and content are separated. Pedagogical topics at the end of course for a bachelor of science. Secondary teachers must be university graduates; in some rural areas they are not.</td>
</tr>
<tr>
<td>France</td>
<td>Poor inservice programs; focus on preparing students for exam</td>
<td>Teacher associations involved in decisions at highest levels All teachers graduated in math at rather high level – 4 years plus one year in teacher training</td>
</tr>
<tr>
<td>India</td>
<td>Unplanned inservice, focus on examination, tutoring as a big business, little link between preservice and practice.</td>
<td>Teachers are respected (in rural areas act as advisors), however, pay for tutors in private sector conflicts Teachers for grade 8 and above are expected to have a mathematics background, but in practice, many have not had much math. There is a strong teachers’ association.</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td>Respect for all teachers Increased pay Very organized difference in preservice and inservice, primary and secondary</td>
</tr>
<tr>
<td>Kenya</td>
<td>Teachers judged by student exam scores.</td>
<td>Specific colleges provide training for primary teachers</td>
</tr>
<tr>
<td>Tutoring is available in urban areas</td>
<td>There is little inservice training</td>
<td></td>
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<tr>
<td>-------------------------------------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>The status of teachers varies by teaching level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A bachelor’s degree is necessary for secondary school teaching</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>Conflicting advice received in teacher education and from colleagues in practice</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beginning to establish a common frame for teacher education at all levels.</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>Beginning of push to teach for exams</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension between how teachers learn content, and how to teach</td>
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</tr>
<tr>
<td></td>
<td>No quality control or uniformity in preservice or inservice programs.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Less focus on examinations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No such thing as private tutoring</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nearly all teachers have university degrees and some mathematics training, although many have provisional licenses.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Status of teacher varies by state.</td>
<td></td>
</tr>
</tbody>
</table>

**Observer Commentary**

_Hiroshi Fujita_

_The Research Institute of Educational Development, Tokai University_

There are three reasons why teacher education is necessary: 1) the people that teachers teach are not mature, 2) teachers have to be more future-oriented than their students or mathematicians. The mathematics that they teach must be of value or use in the future for their students. 3) Teachers must first take care of students and encourage them. In order to have more qualified teachers, teachers must be paid more, be respected by society, and given time to think and learn. Teachers cannot work on their professional skills because they are too busy right now taking care of all aspects of their students.
Hyman Bass  
University of Michigan  
Teacher education seems to consist of some study of mathematics, followed by study of pedagogy, and in some cases, the study of mathematics and teaching is integrated. The separation of these two domains of study poses many problems, perhaps leading to the perception that the study of pedagogy was of no use in practice. In well-designed programs, this should not happen.

The question about what knowledge is most useful to teachers remains an open one. Often teachers learn mathematics as mathematicians teach and learn it, but this form of knowledge doesn’t translate well in classrooms. For example, [as was illustrated in the video of the third grade class struggling with even numbers] teachers need different kinds of understanding of mathematics. This knowledge is specialized, just as engineers, biologists, and economists, need a different kind of mathematical knowledge. We do not, however, have a deep and robust understanding of the mathematical knowledge that teachers need, combined with knowledge of children’s cognition, and so on.

Sweden is developing an interesting program. A shared core program is about one-third of the total requirements for both elementary and secondary preservice teachers. The hypothesis is that there is a certain kind of mathematical knowledge that is common to all levels. It is interesting to consider what aspects of mathematics might form that core - the nature of number and number systems, the uses of mathematical language, uses of symbolic forms, and mathematical reasoning – a course focused on these ideas.

There is a distinction between teacher education for preservice and inservice. There are relatively underdeveloped programs for practicing teachers in many countries, except perhaps in
Japan where many people do lesson study. There are many questions about the best curriculum for professional development.

Themes That Emerged From the Discussion

Theme 1: Connection between preservice teacher education programs and the practice of teaching

Much of the discussion during the session focused on teacher education or preservice training. A common issue was the lack of connection between what took place in many teacher preparation programs and the reality of the classroom. This seems to reflect a mismatch between what prospective teachers are being taught and the expectations and needs of the classrooms. The majority of the group felt teacher education was important, however, offering reasons such as building confidence as a teacher, learning other knowledge necessary for improving teaching, and establishing teaching as a profession.

The group felt that programs should be improved. The total education of teachers needs to be planned, taking into account needs of country and culture, goals of policymakers, and considering teacher education at different levels. There was strong support for the fact that preservice was not enough. Teachers need support and mentors in their initial years of teaching and to belong to networks and working groups. Linking classroom teachers with university instructors can be a promising strategy to enrich the understanding and growth of both groups.

Illustrative Quotes

“New teachers may be advised by their colleagues to ‘forget what they have been taught in faculties of education’.” (Mina)

“This advice that newly educated teachers get in their first position points to a contradictory situation between teacher education and practice in school. This contradiction makes it difficult
for teachers to implement what they have learned and become enthusiastic about it during their teacher education experiences.” (Brandell)

“A good mathematician is not necessarily a good teacher; need some training.” (Francisco)

“Most important is to establish the profession of teaching. So, teacher education is a necessity. But it needs improvement. The preservice and inservice needs to be changed a lot, taking into account present goals and expectations of the society.” (Agarkar)

**Theme 2: Continuing education for mathematics teachers**

The group voiced strong support for the fact that preservice was not enough. Teachers need support and mentors in their initial years of teaching and to belong to networks and working groups. Linking classroom teachers with university instructors can be a promising strategy to enrich the understanding and growth of both groups. Continuing education was considered important for reasons such as helping teachers

- grow as teachers,
- keep abreast of changes in mathematical content,
- learn about new goals and directions for mathematics education,
- understand more about how students learn,
- keep pace with technology, and
- understand how to teach problems based in real world settings.

Many of the eight countries do not have organized inservice programs for the continuing education of mathematics teachers, and in other countries, the programs are not well designed. In many of the countries, no structures exist to support and enable mathematics teachers to interact and collaborate with and learn from each other. A common perception across the countries is that many inservice programs were not part of a larger picture of building teacher knowledge but random efforts with no coordination nor control. It is difficult for teachers to work on their professional skills because they are busy taking care of their students.

**Illustrative quotes**
“When a new teacher is in first years, whatever his training has been, he needs some tutoring and mentoring. If he finds new problems, new difficulties, new questions, he needs someone to refer to. It is not preservice training. It is something that goes a little further in first years of teaching.” (Sackur)

“Teachers need to decide for themselves if they need more mathematics or what it is that they need. That’s why they need the power to decide what is going to happen with their professional lives.” (Lins)

“Inservice training sessions are routine and unplanned.” (Mina)

**Theme 3: Interesting Innovations**

Egypt is using a video conference approach to reach teachers, but the problem of developing programs that can reach all teachers and that will have a follow-up component to enable teachers to continue their growth is enormous.

Sweden is moving from a centralized to a decentralized system of education and consequently is expecting teachers to make decisions, to translate goals into practice, and to adopt changes occurring in the curriculum. Teacher education is considered to be an instrument to influence classroom interactions. Politicians are much interested in this.

**Key Questions**

Key questions about teacher education include those related to:

**Content and curriculum**

- What should the mathematical content for prospective teachers be? What should teachers learn about teaching and about teaching mathematics?
- Who decides the curriculum/syllabus for mathematics teacher education programs? What are the qualifications of people involved in this work?
- Who decides the curriculum/syllabus for mathematics teacher inservice programs? What are the qualifications of people involved in this work?
- How could teacher education programs do a better job of responding to and preparing teachers for realities and constraints of school contexts?
Structure

- What is the role of the university and mathematics educators in the community of teachers? What interface exists between classroom teachers and those at the university?
- What are some promising strategies for promoting more collaborative work and policy-setting between teacher educators and school personnel?
- Who is in charge of ensuring that teacher education programs do what they need to do? What are effective supervision mechanisms to ensure that the objectives of a teacher education program are achieved?
- Who is in charge of ensuring that inservice programs are of high quality, delivering important and useful content? What are effective supervision mechanisms to ensure that the objectives of an inservice program are achieved?

Policy

- Should a country have a standard, uniform system for educating prospective mathematics teachers, or is there an argument to be made for differentiated programs within a country?
- How can programs be put into place to provide inservice for teachers in countries with large numbers of teachers, and little history in this area? How can a country’s educational system deal with uneven communication, and large numbers of teachers who have varying backgrounds, and degrees of expertise?
- In many countries the primary goal of mathematics education is to prepare students to pass national examinations. Given such a goal, what are the implications for teacher education programs for preservice teachers? Should teacher education programs in countries driven by a mandated exam have certain characteristics or components?
- What would be an effective system for the continuing inservice education of mathematics teachers? How could this be realized? What sorts of policies would be needed? Who should be involved in setting such policies? And what should be the role of the classroom teacher?
- How might mathematics teacher education programs be rethought and redesigned to respond to teacher shortages? What are some promising strategies for recruiting more mathematics teachers? Who, ideally, should be recruited?
- How does the permeation of tutoring and private schooling in mathematics interfere with the professionalism of teachers?
Opening Statements

George Eshiwani  
Kenyatta University

Education in Kenya consists of one to two years of pre-primary education (ages 4-6), eight years of primary education (ages 6-14), four years of secondary education (ages 14-19), and four years of university education leading to a bachelors degree. Two major examinations are set by the National Examinations Council; the Kenya Certificate of Primary Education (which takes place at the end of primary school), and the Kenya Certificate of Secondary Education (which takes place at the end of secondary school). Each exam determines whether or not students progress to the next level of education.

In Kenya the curriculum is controlled by the Kenyan Institute of Education, which draws its representation from a wide range of teachers and experts from universities. The secondary school mathematics syllabus is very demanding on the majority of students, many of whom find certain topics extremely difficult to comprehend. Algebra, as well as probability and statistics, are included in the Kenyan curriculum. In the first year of secondary mathematics education, algebra content emphasizes coordinates and graphs and simplifying expressions. In the second year, students encounter linear equations; quadratic expressions and equations; linear
inequalities; and basic statistics\(^{11}\). In the third year of secondary mathematics education, students continue their work with quadratic expressions and equations and are introduced to binomial expansion, matrices, sequences and series, and probability. In the fourth and final year of secondary mathematics education, students study matrix transformations, statistics involving variance and standard deviation, time series and trends, and indexed numbers including weighted averages. In addition to these topics, students in the fourth year also study linear programming, differentiation, and integration.

A number of examples taken from the secondary examination illustrate some of the difficulties students have with various concepts in Algebra, Probability, and Statistics. Two examples are:

1) \textit{Simplify} \[
\frac{2x-2}{6x^2-x-12} + \frac{x-1}{2x-3}
\]

Although this item was targeted towards average students, 80\% of students received a score of 0, and 10\% received the maximum score of 3.

2) The volume \(V\) cm\(^3\) of an object is given by \(V=(2/3)\pi r^3[(1/sc^2)-2]\). \textit{Express} \(c\) \textit{in terms of} \(\pi, r, s,\) and \(V\).

Although this item was targeted towards average students, 80\% of students received a score of 0, and 5\% received the maximum score of 3. To receive 1 point (which was attained by 3\% of students), the students needed to arrive at the following: \((1/sc^2)=[(3V)/(2\pi r^3) + 2]\)

\(^{11}\) Basic statistics involves the use and interpretation of data organization tools such as histograms and pictograms, and the calculation and interpretation of measures of central tendency.
Although the curriculum is well spelled out in the policy statements, student performance was nowhere near expectations. Reasons for this range from poor teaching to acute shortages of mathematics textbooks.

*Beatrice Shikuku*
*Booker Academy*

In Kenya about 10% of the children like and are willing to study math. The rest have to be persuaded or forced to study mathematics because it is compulsory. They have a completely negative attitude towards the subject (especially the girls), and therefore, as a classroom teacher I find that teaching mathematics in Kenya has been, and still is, an uphill task. The main reason for these problems is that up to the late 1970s, nobody chose to go to the university to study education as a profession. The good mathematics students studied engineering, medicine, accounting, or any other course but teaching. Many of those who failed to meet the minimum requirements for their preferred careers became teachers. Such mathematics teachers tended to scare the learners to cover up their lack of content knowledge and their inadequate preparation to teach the lessons.

Children seem to find the learning of mathematics difficult and painful. It took a very bright and brave child to accept the pain and learn mathematics. It was even worse for girls as they often could not withstand the fear. The situation got even worse in the 1980s as those educated in this manner became the educators. In addition, many students came to school having heard horror stories about mathematics learning from their parents. These factors gave mathematics a monstrous face, and to date, we are still trying to change this image to one with a more friendly face.
As one who has willingly chosen to be a math teacher, I have gone out of my way to address my colleagues in an effort to convince them to put a smile on our subject. Since I am also an examiner at the national level, I meet over 400 mathematics teachers once every year, and I believe we are making a breakthrough. We are making mathematics smile. I have also gone out of my way to talk to parents and convince them that mathematics is not as bad as they thought but that it was taught badly. I tell them that the ugly monster of their day is no more, and that it is now very enjoyable to learn mathematics in many schools.

I have made mathematics very popular by eliminating punishment to the slow learning and taking more time with them instead. I always wear a smile on my face, try to be fair but firm, and ensure that they complete their homework. I use real life situations and objects that can be seen and touched during the lessons. As a result, I have registered 100% pass rates in my class, and this has given me the courage to visit other schools in the country and speak to students on how to make mathematics very easy.

In the early eighties the government restructured the mathematics syllabus, which previously had options to take care of varied potentials in mathematics. Now there is a common syllabus for all. Allowing different options of mathematics had a very negative effect on learners who ended up with the option considered to be for weak students, while encouraging arrogance in those who took the option for stronger students. This arrogance developed at an early stage in life and unfortunately spilled over into the teaching of mathematics by those students who ended up being mathematics teachers. The common syllabus used now is appropriately designed to take care of students with different potential. While there are many textbooks, they have the same basic content. The Kenyan Institute of Education approves books, and teachers have a vote
in approval. These steps have helped to improve the image of the subject. Many people now appreciate the value of mathematics.

Also, since March of 2001, the Kenyan government has banned corporal punishment in schools. This should go a long way toward making mathematics acceptable and, therefore, easy to teach. The good news is that in the year 2000, only 12% failed mathematics at the KSCE (secondary) level. Four years ago the failure rate was 38%. This is a great improvement.

Participants identified the similarities and differences represented in Table 4.

Table 4 Kenya: Similarities and Differences Among Participating Countries with Respect to Content in the Mathematics Curriculum

<table>
<thead>
<tr>
<th>Country</th>
<th>Similarities to Kenya</th>
<th>Differences from Kenya</th>
</tr>
</thead>
</table>
| Brazil  | Teacher knowledge strongly influences curriculum enactment  
Curriculum content is similar  
Statistics content involves descriptive statistics and some data analysis  
Similar problems with teachers and teaching approach | Different syllabi for different students. Math is not for everyone.  
Many believe that math is boring and there is nothing teachers can do about that. |
| Egypt   | Algebra is central in secondary curriculum  
Problem is with teaching approach, not with the curriculum |                      |
| France  | All students take algebra  
Gap between expectation and student performance on exams | Most students perform adequately on assessments of algebra, statistics, and probability knowledge |
| India   | Algebra is central in secondary curriculum, which includes statistics  
Students perform poorly on data analysis items  
Instruction is calculation based, not interpretation based | Few students study statistics  
Different curricula for students based on college plans. Non college-intending and non-science |
<p>| Japan   | Algebra is central in secondary curriculum, probability is taught to all students, and most high school students take calculus |                     |</p>
<table>
<thead>
<tr>
<th></th>
<th>Sweden</th>
<th>United States</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher knowledge strongly influences curriculum enactment</td>
<td>students are required to take fewer core courses.</td>
<td>Effort to connect math to real life; instruction is focused on interpretation and meaning</td>
</tr>
<tr>
<td>Effort to connect math to real life; instruction is focused on interpretation and meaning</td>
<td>Teacher knowledge strongly influences curriculum enactment</td>
<td>Students have a negative attitude towards math</td>
</tr>
<tr>
<td>Students have a negative attitude towards math</td>
<td>Most students have a positive attitude towards math (Half to three-fourths say that they like and value math)</td>
<td>Effort to connect math to real life</td>
</tr>
<tr>
<td>All students take algebra</td>
<td>Different curricula for students according to ability. Not all take algebra</td>
<td>Some curricula are strongly interpretation-based</td>
</tr>
<tr>
<td>Algebra is central in secondary curriculum, and statistics and probability are becoming more popular</td>
<td>Statistics and probability not central in secondary curriculum</td>
<td>Teacher knowledge strongly influences curriculum enactment</td>
</tr>
<tr>
<td>Effort to connect math to real life</td>
<td>Calculus not taught to all students</td>
<td>Gap between expectation and student performance on exams</td>
</tr>
<tr>
<td>Some curricula are strongly interpretation-based</td>
<td>Access to texts is a problem in some rural and inner-city areas</td>
<td>Students are required to take fewer core courses.</td>
</tr>
<tr>
<td>Teacher knowledge strongly influences curriculum enactment</td>
<td>According to TIMSS, students value math and say that they like it</td>
<td>Statistics and probability not central in secondary curriculum</td>
</tr>
<tr>
<td>Gap between expectation and student performance on exams</td>
<td>Students do not do as well as expected on exams</td>
<td>Effort to connect math to real life</td>
</tr>
</tbody>
</table>

**Observer Commentary**

*Hiroshi Fujita*

*The Research Institute of Educational Development, Tokai University*

About algebra in the secondary school:
1. Algebra is the main transition from numeracy to mathematics.
2. Algebra is the core language of mathematics. Because of this, many other topics need knowledge of algebra (e.g., probability, geometry).
3. Algebra is a basic component of mathematical literacy.

About the content of algebra in the secondary school:
1. The content is rather stable over time and across countries.
2. The concept of “algebra for all” depends on the enrollment ratio of the age cohort to secondary education. In Japan and in the U.S., the enrollment ratio to secondary education is almost 90 percent, so it is impossible to institute algebra for all overnight.
3. The concept of “core algebra” in high school makes sense.
My understanding of the content of algebra:

- Number systems (rational numbers, irrationals, complex)
- Use of symbols
- Symbolic manipulation of expressions (expansion, factorization up to binomials)
- Binary relations (=, <, >, . . .) binary equations
- Unitary operations (\( x \rightarrow |x|, 1/x, x^n, . . . \))
- Sequential operations

**Hyman Bass**  
*University of Michigan*

We talked about algebra, statistics, data, but most of our conversation was about algebra.

One issue was “what is algebra,” and another was “why do we teach algebra”? Lins said that up to algebra, we teach math of the street, but algebra is not the math of the street. Hiroshi made an interesting assertion with which I am not sure I agree. He said algebra for all is maybe idealistic in countries where all students are in secondary education. Is algebra essential enough to teach all students? I believe that it is both relevant and possible to do so. But we will have to see over the next few decades.

The curricula of the different countries were remarkably similar except for the U.S. Here algebra has been reinterpreted. The portrait of algebra from most of the countries is like what Professor Fujita put on the overhead. In the U.S., there is a movement to give greater emphasis to the notion of function as the unifying concept. Function has a distinguished history in mathematics. Felix Klein said this at the turn of the century. Lins said it doesn’t matter. He is right—this is a matter of convention and definitions.

Recall the video when Shea spoke of the number 6 being odd. The tension in that discussion derived from the fact that kids were using the same term in different ways. It became necessary to reconcile the different definitions. Like those students, we better have a common
definition for what we mean by “algebra.” We should recognize that we mean different things by “algebra.” We might need more terms.

The rationale is that in the modern world, people are inundated with data and have to process quantitative information, and so some notion of patterns, functions is appropriate in the school curriculum. To do this takes a lot of time, time that is taken away from a traditional emphasis on skills and notation. Rational expressions are the expressions needed in calculus. If students enter the university and cannot work with them (the expressions), it will be a problem. But we need a more intrinsic rationale. Many students are not going to go on for more mathematical study.

I think we should ask what the algebra of the street is. Why do we require kids to study so much mathematics for 12 years? Many of us are convinced that mathematics enables people to function more rationally as adults, but it is difficult to specify this in ways that would define a curriculum. It is hard to make a direct connection to everyday life. But without this, adults are impoverished in everyday life. For example, in algebra one learns to name things. If you wrote a novel you would not call all the characters he and she and he and she. The act of naming things, giving them a compressed expression, is central to algebra so that you can manipulate them more efficiently, and communicate more clearly.

We tend to think of learning mathematics as learning some particular topics, but mathematics offers learning beyond its specific topics. I am not prepared to argue that some particular topic has to be in the curriculum. Lists of topics are easy to make. The impediments to children learning these lists is that there are things to know to do, certain habits of mind that mathematicians do. But these things are not explicitly taught. When instructors ask students to
prove something, this is a highly developed skill, and we assume that children can just do it.

Mathematical practices are just as important as topics, and we don’t teach them except abruptly. We need to make mathematics practices an integral part of the curriculum, even in the early grades. Algebra is a subject where many of these practices arise.

Themes That Emerged From the Discussion

Theme 1: The mathematical content of the curriculum

A central theme that arose during this session was what considerations are important in making decisions about mathematics content in the school curriculum and syllabus. In particular, the group focused on algebra, statistics, and probability and attempted to spell out the purposes for requiring students to learn these subjects. Some important factors to be considered in selecting mathematics content include helping students develop certain “life skills” that will enable them to become informed citizens, empowering them to study further mathematics, and preparing students for examinations. Discussions also highlighted the tension between learning mathematical skills just for the sake of skills versus learning mathematical skills for the purpose of applying them to situations and contexts.

Illustrative quotes

“In the U.S. we have the same questions about the content of algebra as that described by the Kenyan curriculum. We have been trying to move the foundation of some of the algebraic ideas down to primary grades. The new Standards push ideas about patterns down to second grade. We have been more successful in developing data and statistics than we have with probability. Our students don’t understand it. Except for simple things, they have trouble. We separate combinations and counting problems away from probability.” (Burrill)

“In India the algebra content is pretty good. There is more than what is expected from a traditional curriculum. Most schools have average students comfortable with algebra, but for below average students algebra is one of the great stumbling blocks. Algebra, logs, and proofs are the three big stumbling blocks for below average students. Word problems are also difficult.
Those preparing for the competitive exams have high level of skills. Student performance in statistics and probability is very good because it is algebraic in nature. In data analysis their performance is poor. Their computations are not based on valid interpretations—not on what things actually mean. Another area of weakness is the visualization in the curriculum (e.g. reading and interpreting graphs, and using guide maps).” (Shirali)

“Many adults find algebra relatively useless and don’t see what it is good for. Our parents and our public don’t see the value.” (Burrill)

**Theme 2: Assessment/examinations and inadequate student performance**

National exams were seen as having a strong influence on teaching. In many countries, teachers’ effectiveness is based entirely on how well their students do on exams. A common concern was that the exams do not necessarily reflect student understanding or their true levels of achievement.

**Illustrative quotes**

“The results of exams especially in developing countries might not have anything to do with actual level of student achievement.” (Mina)

“In France there is a gap between official expectations of students and results. We cannot tell what students are really able to do. Exam results very often do not mean anything. When we try to assess true learning, we also find gaps, but most exam items relate to skill testing. Where do you try to learn to teach students how to use the skills in resolution of situations? We should teach basic skills, but students should learn to use them in context.” (Bodin)

**Theme 3: Algebra – its purpose, content**

In general, algebra as it is reflected in the syllabi of the various countries was similar across the countries. Algebra tends to include equations, expressions, binomials, and complex numbers, and serves as a tool for modeling, and expressing relationships in symbolic forms. In most countries, function and related concepts are not considered as part of algebra but rather of analysis.
Illustrative quotes

“Many people said they had some of the same topics in your curriculum. Algebra is a very big subject. For example, Hashimoto said that they treat cubic equations in Japan. What about polynomial functions of various degrees – linear, quadratic, and higher order? Are general theorems about polynomial functions included—for example, the binomial theorem? Most of you made reference to exponential and log. Do you have the natural e^x? Suppose we only consider 2^x? This is introduced first for an integer variable x. Is it treated when x is a continuous variable and, if so, how is it done? What is the boundary of this subject in the curriculum?” (Bass)

Theme 4: Motivating students to learn mathematics

Different strategies have been used across our countries to motivate students to learn mathematics. High stakes national examinations that have serious consequences for students are one means. Sometimes in the past, punishment has been used. Making mathematics interesting, meaningful, and useful to students was seen as a way to motivate their learning.

Key Questions
Key questions that arose during the discussion related to:

The nature of the content

- Should students learn mathematical skills just for the sake of skills? Or should the emphasis be on applications to the real world?
- Given the power of technology today, what mathematics is important for students to learn? What mathematics should become obsolete?
- Can algebra be thought of both as basic skills and also basic skills that can be used in context?
- What level of manipulation of symbols is necessary in algebra, and what is the role of technology in general?
- What is really important for students to learn and know in algebra?
- What are effective strategies for determining the similarities and differences in the algebra (or probability or statistics) content that students are taught in different countries? Is it enough to compare lists of topics? To compare examination items?

Relationship between policy and content

- How do educational policies send messages about what mathematical content is valued? And how is this translated into practice?
• How can educational policies effectively address equity issues, such as accessibility to technology, that impact mathematics instruction?
• Have the purposes of learning algebra been adequately established and is there consensus (among policy makers, teachers, the public) about this? Why is algebra in the mathematics curriculum?
• Why is algebra a big stumbling block for below-average students?
• Do assessments actually reflect what students learn?
**Opening Statements**

*Yoshihiko Hashimoto*

*Yokohama National University*

Changes in mathematics education are currently taking place in Japan. The school week is changing from six days to five days per week to allow children to develop “competency for positive living.” Changes in Japan seem to occur approximately every ten years, although the mathematics content over the last 30 years has been relatively stable. Reform is based on tradition and varies according to the times. Since 1994, Japan has had a “core and optional modules” model for upper secondary school mathematics. An overall objective of the entire curriculum is to foster students’ abilities to think mathematically. “Open” methods of teaching – open process, using different ways to solve a problem; open-ended where problems have multiple correct answers; and open problem formulation where students pose new mathematical problems can help meet this objective. Traditionally, one important feature of learning mathematics was to develop the ability to calculate rapidly.

*Miho Ueno*

*Tokyo Gakugei University Senior High School*

The goal of mathematics education might be seen as learning the basic idea of calculus by the time students graduate from high school. Beginning calculus is taken by 82 percent of high school students in a traditional mathematics program. It is often difficult to implement a new
course of study due to factors such as a lack of teachers in a small school system, which limits
the courses offered to those needed for college entrance, or to a lack of technology. Thus, the
intended curriculum may not get implemented.

The results of the Third International Mathematics and Science Study (TIMSS) indicated
that Japanese students disliked mathematics. The new course of study addresses this by stressing
mathematical activities aimed at helping students appreciate the importance of mathematical
approaches and ways of thinking. What matters is finding principles in given phenomena in the
world and finding materials that will allow students to use mathematics spontaneously. Reform
should be realized through teachers’ attempts to enrich the contents of the prescribed curriculum.

Participants identified the similarities and differences represented in Table 5.

<table>
<thead>
<tr>
<th>Country</th>
<th>Similarities to Japan</th>
<th>Differences from Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Open-ended problems are not explored much in the curriculum.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Teaching approach not influenced by policies to reform schools.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Teaching is teaching, and direct method is used.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tradition relates to how schools approach failing students. Before if a student failed one subject they repeated everything. Now, things are changing. Teachers and parents are unhappy about this</td>
<td></td>
</tr>
<tr>
<td>Egypt</td>
<td>Curriculum recommends open-ended problems</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>Reform tends to occur at 10 year intervals</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reform encourages the use of more concrete activities and more student participation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time in school is also being reduced (from five days to four days per</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Traditional math is very formal and abstract. Does not emphasize computation</td>
<td></td>
</tr>
</tbody>
</table>
India

Traditional math focuses on quick computation without technology. Arriving at solutions, not interpretations. Despite reform efforts, traditional approach still common

Open-ended problems are not explored in the curriculum. One specific method allowed on exam items, but since 1986 there has been more of an effort to promote training in logic, mathematical thinking, and mathematical reasoning

Kenya

Emphasis on teaching high level mathematics

Do not have core and optional program

Sweden

Reform tends to occur at 10 year intervals (since the 1960s)

Changes to classroom practice proceed slowly

United States

Traditional math focuses on closed problems, and quick computation without technology

Despite reform efforts, traditional approach still common

Some curricula emphasize open-ended problems and flexible thinking

Reform tends to occur in cycles

Changes to classroom practice proceed slowly

Students say that they find math enjoyable (TIMSS)

Observer Commentary

Hiroshi Fujita
The Research Institute of Educational Development, Tokai University

Traditional mathematics, particularly ancient mathematics should be appreciated as a component of the culture of the country, and including these topics in the curriculum is a good motivation for learning. When considering traditional mathematics for inclusion in the curriculum, the value of the traditional mathematics must be examined from the perspective of the purpose of the curriculum. Origami has never been explicit in the mathematics curriculum in

Mathematics Education Around the World: Bridging Policy and Practice. Reflections from the 2001 International Panel on Policy and Practice in Mathematics Education
Japan; it is regarded as a child’s skill, for fun. When the Japanese government needed to establish the education system in Majii Restoration, they chose to go with western mathematics. The Japanese *Wasan* lost its connection with science and mathematics and was not clearly part of the curriculum any more.

It is not always clear what tradition means within a culture. Tradition and its relation to the existing curriculum can be examined from the perspective of purpose, process, and strategy for reform. One philosophy in Japanese education was to cultivate mathematical intelligence in students with foci on mathematical literacy and mathematical thinking, in a proper balance. The core and option structure of the mathematics curriculum was part of a strategy intended to make depth and breadth compatible.

Mathematics education can be compared to medical science. Mathematicians do mathematics for its own sake. The study of mathematics education is for the practice of teaching. Teachers correspond to practical physicians. Research on mathematics education is similar to basic study of practical education. Researchers must be trusted by practitioners. The role of mathematicians is to help teachers and not to criticize.

*Hyman Bass*
*University of Michigan*

Tradition can be interpreted two ways: in a cultural sense and in a sense of habit. We heard interesting cases from Japan and India of ancient mathematical traditions. Some of this is echoed in work on ethno mathematics, where people in various cultures attempt to identify attention to mathematical ideas deeply embedded in culture, sometimes explicitly, sometimes through art and other cultural artifacts. It is important to honor those traditions.
In some countries, opposition between tradition and reform is concerned with the ways we have been teaching mathematics over the recent past. At times, either the teaching or learning is found to fall short of expectations, or the needs of society cause new demands. This is when change is attempted. A typical reaction seems to be to find fault with the old system, discard everything that was done in the past, and replace it with something new. This is at the root of debates about basic skills. Frequently what seems to be wrong about teaching basic skills is not so much with subject matter but with the method of instruction. Typically in the U.S., to learn computation children would be given formal rules and pages of exercises. The conclusion was drawn that teaching these mathematical ideas implied a kind of teaching and pedagogy that led to an oppressive experience. The result was to remove emphasis on these parts of mathematics from the curriculum. Opposition between tradition and reform is somewhat artificial. Change should be made carefully, and serious thought given before abandoning topics.

Themes That Emerged From the Discussion

Theme 1: Patterns of reform

Policies mandating or supporting change can help or hinder actual classroom implementation of the ideas underlying the changes. Changes in curriculum seem to occur in a cyclic fashion, in some instances approximately every ten years. In some cases, such as Japan, this is part of a designed reflection on the status of mathematics education; in other countries these changes occur for reasons ranging from a change in society’s structure to advances in technology that influence mathematics education. The group concluded that societal demands drive curricular change and determine the nature of the change, including changes in the way mathematics is taught as well as changes in content. For example, some countries structure their
mathematics curriculum using a spiral approach, with a topic revisited each year over several years to develop a deeper understanding of the important ideas.

The role of professional education organizations varies considerably. Some countries, such as the United States, have strong mathematics education associations, and members of these groups are involved in the reform process from the beginning. In other countries, such organizations have yet to be formed or are just beginning to take active roles in thinking about changes in mathematics education.

The group agreed that to make change successful all members of society had to be part of the process. Some reform initiatives involve changing the time allocated for school, which affects the time allocated for mathematics. Japan and France are both shortening their school week; some secondary schools in the United States are redesigning the internal structure for the time allotted to specific content areas. Brazil, to accommodate students who work, has a system of three days of school and three days of work.

Illustrative quotes

“We try something for a year or two, and if it doesn’t work we throw it out and start over. For example, we had modern math – no good, threw it out; then back to basics – no good, threw it out.” (Burrill)

“New curricula are also introduced about every decade in Sweden – from 60s and after, foci are very much the same as in some other countries. It is interesting that this seems to be so worldwide.” (Brandell)

Theme 2: Appreciation of mathematics

Policies and programs are needed to link content and practice in classrooms in ways that build an appreciation for the use and power of mathematics. The attitude of students towards mathematics was a common concern. In most of the countries, society at large feels that
mathematics is important and supports the role of mathematics as an integral part of the
curriculum. It is acceptable, however, to be mathematically illiterate or to proclaim that you are
uncomfortable with mathematics. Such an attitude influences students in classrooms and
interacts with what teachers do as they teach. This negativism may be due to the topics that are
taught or to the way the mathematics is taught. Educators in all countries are searching for
approaches that move from punishing students for not succeeding to finding ways to engage and
support them as they learn.

One strategy for making mathematics more attractive to students is to emphasize changes
in the practice of learning mathematics rather than changing the content (Sweden). Another
approach is to link content and activities to make the topic interesting for students (Japan).
Another is to link some of the historical elements of a society to the mathematics curriculum.
For example, the tradition of Vedic mathematics, rooted in the Indian culture, can help children
do calculations quickly. In the United States, some teachers and curricula advocate working
from data as a way to motivate students.

Illustrative quotes

“You just don’t meet math outside, so either you like it in school or not. History, etc., you meet
outside. Maybe pupils want to play mathematics and not do the drill. It’s the same problem as
faced by those who train soccer players—how to get them in shape without the drill.” (Lins)

Theme 3: The role of tradition in reform

Tradition can be used as a platform for improving mathematics education or it can be a
hurdle. Each country has a history of mathematics education that has transmuted in various ways
into current policies and practices. “Traditional” mathematics is relative and means different
things to different countries. In some countries such as France and the United States, tradition
means relatively formal and abstract content and teaching. To Japan and India, tradition refers to early mathematical techniques and processes such as Wasan or Vedic mathematics. Tradition to some can also mean strong expectations for certain ways of teaching and for learning certain content based on what was done when the current adults were in school.

Part of the tradition of mathematics education is who has responsibility for student successes and failures. In countries such as Kenya, Brazil, and India, students are responsible for their own success or failure. In the United States, France, and Sweden, failure is not seen as the failure of an individual child but rather as a failure of the educational process.

Illustrative quotes

“We don’t have a strong tradition about certain ways of doing things. We have strong expectations among parents, students, and school administrators about what math should be and what students and teachers should be doing. We are stressing open problems, but the word open can bring lots of confusion. We do not necessarily mean problems that do not have solutions. We mean that there are lots of ways to think about the problem and move towards the answer.” (Burrill)

“Vedic mathematics is a collection of elegant computational algorithms, but it can be inflated beyond reasonable dimensions as a means to solve all problems in mathematics, like Fermat’s theorem.” (Shirali)

Key Questions

Key questions about tradition and reform related to:

Motivation and nature of reform

- Who is in charge of reform? What mechanisms are used to ensure that the principles of reform are clearly understood and implemented by the entire system?
- Does the system have the capacity to implement reform?
- What steps are taken to ensure that examinations, including university entrance examinations, are aligned with the directions of reform?
- What is the role of teachers in bringing about reform?
- How can some of the rich traditional mathematics be incorporated into the curriculum?
- What is meant by tradition? Traditional content or traditional teaching? What is meant by
curriculum and syllabus? Reforms for whom? Teachers, students, ministry of education?

Implementation

- What strategies can be implemented at the policy level to support a changing attitude towards mathematics?
- How do teachers come to know and understand the changes suggested as a country works to improve mathematics education?
- What process is used to ensure that those at the university and those who teach teachers understand the new directions and emphases?
- How can teachers access methods and materials to make mathematics more interesting?
- Will changes in teaching practice enable students to be more comfortable learning mathematics?
- Is tradition a hurdle to modernization?
Issue 5: Depth and Breadth in the Mathematics Curriculum

How does your educational system decide the balance between depth and breadth, that is between insistence on in-depth knowledge of relatively fewer core topics vs. a broad inclusion of topics, with less emphasis on each? How is this decision effected in practice?

India: Sudhakar Agarkar and Shailesh Shirali

Opening Statements

Sudhakar Agarkar
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research

India has a long tradition of mathematics education. Vedic mathematics was an essential part of the education system in the ancient Indian Gurukul system. Mathematics education was, however, limited to a certain class of the society. In colonial times, class education continued and emphasized certain arithmetic topics. The majority of the people had no opportunity to study formal mathematics. Folk mathematics, however, enabled them to handle day to day transactions. After independence in 1947, the focus shifted from class education to mass education, and the school system expanded rapidly. This resulted in a shortage of math teachers with the level of content knowledge needed to do justice to depth in mathematics education.

Education is the responsibility of the states. Different states have different ways of organizing the system. The Education Commission Report of 1966 recommended a uniform 10+2+3 arrangement across states. It also recommended teaching mathematics on a compulsory basis up to grade ten. The expectation of math education was that the courses would be taught at a level high enough to provide the base necessary for advanced study in later years. In 1968 the government of India adopted the recommendations of the Education Commission. The NCERT prepared the curriculum and textbooks. They were criticized as being “heavy” and a two-level
curriculum was suggested but not implemented. There was a high failure rate on the school-leaving exam. Dedication to the concept of mass education forced the preparation of a “lighter” syllabus. In 1986, the National Education Policy (NEP) was adopted. This policy suggested a shift in emphasis, away from manipulation and towards visualizing math as a vehicle to train pupils to think, reason, analyze, and articulate logically.

As the system of mass education was expanding rapidly, two attempts were made to achieve depth. These included: 1) the inclusion of challenging exercises, and 2) the establishment of minimum levels of learning (MLLs). Many teachers and policy makers believed that focusing on depth would eventually pay off as students would be able, as a result, to handle superficial topics on their own when the need arose. Despite this, the prevailing approach was to cut the depth and add breadth to the curriculum.

Another issue that arose out of mass education was the need to provide the necessary knowledge and skills for students with different plans for the future. The National Curriculum Framework in 2000 states:

While determining the curriculum in mathematics, it must be kept in mind that the majority of pupils would leave education at the end of secondary stage (grade ten). They would need to apply math skills and competencies in their work situation. Only a small number would go on to higher education. The curriculum needs to strike a balance between the requirements of both groups.

The framework also suggests that the history of mathematics, with special reference to India and the nature of mathematical thinking should find a place in the curriculum, and students should be encouraged to enhance their computational skills by the use of Vedic Mathematics.

Shailesh Shirali
Rishi Valley School
I teach in Rishi Valley School, a co-educational residential school belonging to the Krishnamurti Foundation of India (KFI). It is located in a hilly and drought-prone region of southern India, and it caters to students ranging in age from 8-17 years (standards 4-12). It is comparatively small in size, with 350 students in all. I have been at this school for nearly two decades, and chiefly teach mathematics at the 11th/12th standard level, and (occasionally) physics and computer science; on occasion, even geography! In addition to my regular teaching, I am closely involved with the Mathematics Olympiad movement in India, and I also do a lot of expository writing (articles as well as books). In this note I shall make a few observations on the teaching of mathematics in India, based on my personal experience—that of teaching at the 11th/12th standard levels.

**Brief note on the Indian education system**

At the 10th standard level, students in India take their first major public examination, in which they are tested on a wide variety of subjects (English, a second language, mathematics, physics, chemistry, biology, geography, history/civics, plus an elective). After clearing this examination they do the “Plus-2” course (11th and 12th standard), and it is here that they start to specialize. They are required to take four elective courses, plus Compulsory English. Here are some typical combinations for which students opt:

- mathematics with physics and chemistry, the fourth elective being computer science or life science or economics;
- biology with physics, chemistry and geography;
- humanities (literature, history) with two electives from mathematics, economics, geography and fine art;
- commerce, accounts and economics, with mathematics or geography as a fourth elective.
There are, unfortunately, several different examination boards in the country: two all-India boards—Council for the Indian School Certificate Examination (CISCE) and Central Board of Secondary Education (CBSE)—plus separate boards in every state. Each board has its own sets of syllabi, and the different syllabi do not mesh particularly well. An unfortunate fall-out, which sometimes takes place because of this multiplicity of boards, is grade inflation.

**Areas of difficulty: personal observations**

There are very strong socio-economic pressures on students to register for mathematics at the Plus-2 level. As a result, enrollment is high. Often one sees students in the class who do not possess the requisite aptitude for the subject. This results in wide differences in ability in the class, and classroom teaching begins to lose its effectiveness. Closing the door on such students is not a satisfactory answer, as they have their future career at stake. For instance, those who wish to study economics are required to do mathematics at the school level. The net effect is that teaching becomes much harder, as individualized instruction is difficult to accomplish, given the pressures of completing the syllabus.

Also, the Indian education system seems to be more examination-driven than any other country. The examinations include not just the school-leaving public examinations but also admission entrance examinations conducted routinely by institutes offering courses in engineering, medicine, law, architecture, management, computer applications, fine art, etc. These entrance examinations are taken by a very large number of students, and pressures for admission are very intense. Inevitably, private institutions (“coaching centres”, “tutorial centres”)

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12 Available choices for electives include Fine Art, Economics, Computer Science, Classical Music, Environmental Science, Home Science, and so on.
14 A copy of the CISCE syllabus for Mathematics Classes 11 and 12 is in Appendix. My own school is affiliated to the CISCE.
colleges”) have sprung up over the country, offering professional help (at a stiff price, of course) for examination preparation. This in turn has led to a general rise in the level of sophistication of the examinations, leading to a still greater demand for tutorial colleges, and so on. The cycle is a vicious one, and becoming steadily more vicious with time. The obvious fallout for students is that they are divided between competing demands. The syllabi for entrance exams do not completely mesh with their regular school syllabi, and the styles of examination also differ in many ways. The effect of such stress on motivation may easily be imagined. Another inevitable fallout is that teaching tends to focus on training rather than on education. Without question, the examination-driven nature of our educational system has had a serious negative effect on the country.

For further consideration of the question of depth versus breadth through four case studies, see Appendix E.

Concluding remarks

As mentioned earlier, I have drawn from my own experience in writing the above comments. However, I have also received some comments from other teachers of mathematics. For instance, a colleague who teaches at the 10th standard level reported to me that “the syllabus seems to be very wide but not too deep”. The comments that I made, independently, are about the syllabus for 11th/12th. The example he quotes is that of the unit on commercial arithmetic, wherein students are expected to acquire an understanding of insurance schemes, taxation (personal income tax as well as sales tax), shares and dividends, etc. Much of this material lies totally outside student experience and, thus, is thrust on them, and of necessity the coverage is very superficial. Other examples may be given. Pythagoras’s theorem is studied but only its
It may seem that I am highlighting only the portions of the syllabus with which I have some quarrel. This is certainly so! However, it is not as though I have such feelings for every segment of the syllabus. Fortunately there are many portions that receive fairly good coverage: coordinate geometry in two and three dimensions, differentiation, integration, differential equations, vector algebra, complex numbers, determinants, etc. (Interestingly, these are all traditional “old-fashioned” topics; their coverage in high school/college must have been much deeper during the second half of the 19th century and the first half of the 20th century.) Here the depth of coverage is adequate, and students tackle a variety of problems.

I should add here that I have often attempted to go “beyond the syllabus”. For instance, in the group theory segment I have brought in the mathematics of the Rubik cube, and in the segment on the mean value theorem, I have shown how these results help in arriving at some nice inequalities [e.g. for the sine function, or for the function: square root of (1+x)]. These initiatives have, however, received mixed responses. This is discouraging at first encounter, but one must recall to oneself the tremendous pressures which students face because of the public examination.

The more traditional areas receive good coverage (see sample exam questions in Appendix D). However, much more could be attempted, and that is what I have tried to focus upon in this essay.

Participants identified the similarities and differences represented in Table 6.
Table 6 India: Similarities and Differences Among Participating Countries with Respect to Depth and Breadth in the Mathematics Curriculum

<table>
<thead>
<tr>
<th>Country</th>
<th>Similarities to India</th>
<th>Differences from India</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>There is more breadth than depth in the curriculum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Students from wealthy families have more educational options</td>
<td></td>
</tr>
<tr>
<td>Egypt</td>
<td></td>
<td>Curriculum has a great deal of depth, but it is disconnected/fragmented</td>
</tr>
<tr>
<td>France</td>
<td>Teachers find it difficult to cope with curriculum and go deeply.</td>
<td>Expectations are determined locally</td>
</tr>
<tr>
<td></td>
<td>Minimum tends to become the maximum. The notion of challenging problems is difficult</td>
<td>Teachers modify the ways they approach a topic</td>
</tr>
<tr>
<td></td>
<td>to get into curriculum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exam-driven; exam at the secondary level is made by an exam board,</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>Curriculum has less depth than it used to</td>
<td>Teachers cannot modify the national curriculum, but they can adjust the breadth and</td>
</tr>
<tr>
<td></td>
<td>There are minimum competency levels</td>
<td>depth of coverage for mandatory topics</td>
</tr>
<tr>
<td>Kenya</td>
<td>Math is also compulsory at least up to year 10</td>
<td>Exam results determine which students proceed past year 8</td>
</tr>
<tr>
<td></td>
<td>Students must pass school-leaving exam</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Teachers teach to the tests</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>There are minimum competency levels</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Math is also compulsory at least up to year 10</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>There is also a tension between breadth and depth</td>
<td>Districts can modify the state curriculum/standards</td>
</tr>
<tr>
<td></td>
<td>Depth heavily influenced by teacher content knowledge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Many states are adopting school-leaving exams</td>
<td></td>
</tr>
<tr>
<td></td>
<td>There are many perspectives and sets of standards</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exams often do not mesh well with</td>
<td></td>
</tr>
</tbody>
</table>
Observer Commentary

Hiroshi Fujita
The Research Institute of Educational Development, Tokai University

Depth is appreciated if it helps to enhance students’ mathematical thinking (creativity and heuristic insights included.) Gifted students are exceptions; let them go higher. A small number (about .0005%) will make good mathematicians. Their mathematical thinking power could be acquired by learning theoretically advanced topics. The role of the teacher is difficult; it is the teachers’ role to locate these gifted students, work with them, and get mathematicians to guide them.

Depth in math education is different. For general students, the top 20 percent and majority, mathematical thinking power should be enhanced through challenge against properly hard problems, through problem solving, and open-ended problems.

Hyman Bass
University of Michigan

Those here seem to agree that depth is better than breadth; it is better to probe deeply into a selected set of topics and problems, rather than superficially do several topics. This also seems to be the opinion among accomplished teachers. TIMSS characterized the U.S. curriculum as “a mile wide, inch deep.” In Japan, a country that performed well, typically an entire class period would be used to focus on one or two problems. Why do we not practice instruction this way? One guess might be the role of assessment. Higher order skills are difficult to measure with short items on exams. When you want to implement assessment on a national scale, there are questions of cost. The technology of assessment leads to having great confidence in what is
being measured; the whole methodology of assessment points toward measuring many things in shallow ways. Assessment pushes curriculum toward shallowness. One solution is to better integrate the communities of classroom practice and assessment experts—then assessment could better support the curriculum.

Themes That Emerged from the Discussion

Theme 1: The meaning of depth

The notion of depth in the learning of mathematics can be interpreted in many ways. Depth can mean detailed study of many problems at different levels. Depth might be about the ways that students address the problems we give them. Depth could mean knowing more and more about the subject. Or, it could mean being able to use ideas in relation to other ideas and to understand ideas this way. The group agreed that to know deeply means you know more and more about the subject and are able to use it to solve a different problem. Deep thinking is different from deep knowledge. When you have a deep knowledge, you understand it in relation to other mathematical ideas. To know something deeply is to feel empowered by that knowledge

Illustrative quotes

“The issue of “depth and breadth” have different meanings, which are associated with different thoughts regarding math and math ed. It could mean having details and different levels of problems (the common “traditional” understanding).” (Mina)

Theme 2: Factors related to achieving deep understanding

Assessment can have an impact on the nature of the curriculum and on the depth of understanding expected of students in their learning of mathematics. Depth can be assessed with different types of questions, but often exams have “busy” questions. It is difficult to have
challenging problems on a national exam because of issues such as scoring, finding the problems in the first place, managing the administration of such tests. In addition, different boards in the same country may have different standards.

One way to achieve depth might be by integrating knowledge, but integration of knowledge is not done by students. They have many separate ideas but need help to integrate them. A spiral approach might also help students look more deeply into a concept. Another way is to consider topics that are not necessarily treated deeply but that can be such as the long division algorithm. How do classroom teachers treat an idea? Bass noted that the aim of education should be to have students learn about mathematical practices or to think mathematically or deal with knowledge by themselves. Is the question of depth a consequence of the way the students address certain problems that we give them. Do we give students questions about which they have to make conjectures and look for connections to other knowledge with the depth at the end of the process rather than at the beginning? If we go into depth, we should have a reason to do so, and suggest ways to deepen the ideas.

Illustrative quotes

“It is difficult to reach depth in a spiral curriculum, because the tendency is to repeat the same topics at the same level each time around.” (Shirali)

“We might think of depth as a “helix”- a sort of spiral in three dimensions.” (Hashimoto)

“The minimum tends to become the maximum.” (Bodin)

“A spiral curriculum makes sense only in a setting where you are certain that students will continue their education. If you are not, then decisions about what mathematics must be covered take on a different significance.” (Mina)
Theme 3: Balancing breadth and depth

When a country is committed to education for all, the depth-breadth challenges become even more complex. In many of the countries the curriculum contains far more topics than teachers can possibly cover. In the U.S., each of the 50 states writes its own curriculum; national standards such as the NCTM standards are available as a guide but there is no requirement that they be followed. This phenomenon of rewriting and adapting the national curriculum is not widely practiced in the other countries represented; and when it is, there are still constraints (e.g., in India, states can modify the national curriculum but they must use 75 percent of it.)

Teachers find it very difficult to contend with a crowded curriculum and to also aim for depth. The result is that they can only do minimal treatment of many important ideas. Teachers may think students do not have a good background, and so they teach to lower expectations. They decide to have fewer topics but still do not teach with depth. The classroom teachers in the seminar tended to favor depth over breadth.

Illustrative quotes

“What is the balance between depth and breadth - which is more important - depends on the objectives.” (Hashimoto)

“If the teacher does not know what to do with the student’s question, the teacher won’t go deeply. A teacher can go deep and have students go deep only if the teacher knows the subject deeply.” (Sackur)

Key Questions

Key questions about depth and breadth related to:
**Examination systems**

- Does the inclusion of challenging exercises on examinations ensure that teachers will strive for depth?
- Are topics not on the examination covered in any depth and what is the motivation to do so?

**Curriculum choices**

- How are choices made for moving to fewer topics, and who makes them?
- When teachers are faced with curricula that include too many topics, and they must make choices, on what mathematical knowledge do they need to draw?
- If we put the idea of fostering mathematical practices at the foreground, what kind of meaning does depth then take on?
- Is the question of depth a consequence of the way the students address certain problems they are given?
Issue 6: Excellence and Access

How does your country and culture deal with the challenges of excellence and accessibility in mathematics education? What is the balance of power and input into the system between the various educational constituencies?

Sweden: Gerd Brandell and Susanne Gennow

Opening Statements

Susanne Gennow
Danderyds Gymnasium

Before explaining how we deal with excellence and accessibility in mathematics education, I would like to describe some aspects of the larger school system. As shown in the table below, the Swedish school system consists of pre-school, compulsory basic school, and non-compulsory school (See Table 7 below).

Table 7. School System in Sweden

<table>
<thead>
<tr>
<th>Type of School</th>
<th>Description</th>
<th>Enrollment in 1999/2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-school</td>
<td>Municipalities must offer pre-school programs, but parents voluntarily enroll their child(ren). There are two types of pre-school programs: Pre-school Activity and Pre-school Class. These schools may be public or private. Pre-school Activity programs serve children between the ages of 1 year and 5 years. These programs are primarily non-academic. Examples of preschool activity programs include family day-care and open pre-school programs. Pre-school Class programs serve children at age 6. These programs have a stronger academic component.</td>
<td>Approximately 350,000</td>
</tr>
<tr>
<td>Compulsory Basic School</td>
<td>Children between the ages of 7 and 15 are required to attend school. 97% of pupils attend public</td>
<td>Slightly more than 1 million</td>
</tr>
</tbody>
</table>
schools, and 3% attend independent schools. Those between the ages of 7 and 12 attend Primary School (years 1-6), and those between the ages of 13 and 15 attend Lower Secondary School (years 7-9).

Non-compulsory School

Non compulsory school consists of Upper Secondary School (years 10-12) and Municipal Adult Education.

Upper Secondary School serves students between the ages of 16 and 20. 98% of all 16 year olds enter Upper Secondary School. 96% of pupils attend public schools, and 4% attend independent schools.

Municipal Adult Education serves students over the age of 20 who wish to complete Basic or Upper Secondary education.

In Sweden, national goals govern all subjects in Compulsory School and all courses in Upper Secondary School. These national goals are defined according to two types of objectives—aspiration objectives and objectives that must be achieved. While aspiration objectives describe the qualities of pupil attainment that the schools must seek to develop, objectives, which must be achieved, are those all pupils must be provided with the opportunity to attain. These objectives frame efforts to address issues of excellence and accessibility in the Swedish education system.

Mathematics Education in Sweden

A primary method of promoting accessibility in mathematics education involves requiring that all students in compulsory school take mathematics. By the end of year nine, students have completed a total of 900 hours (60 minutes at a time) of mathematics education. A primary method of promoting excellence involves setting standards and defining goals for student achievement. The standards that relate to mathematics education set goals to attain in years five
and nine. Progress toward the achievement of these goals is measured by national tests. While the national test that students take in year five is optional, in year nine students are required to take the Compulsory National Test. The results of this test influence admission to Upper Secondary School.

Students are eligible for upper secondary school in a national or local program if they attain the degree goals set for Swedish, English, and mathematics. Ninety percent of all students attained that eligibility in 1999. This number is steadily decreasing, though. It was less than 90 percent in 2000. Despite this, 98 percent of students entered upper secondary school in 1999/2000. This 8 percent difference is due to policies designed to increase access to upper secondary education. This policy makes it possible for students who fail to attain the degree goals, to still enter upper secondary school. Students who gain access in this manner follow an individual program where they complete their basic education and in parallel take courses within a national or local program.

Once admitted to upper secondary school, students enroll in a program consisting of various subject courses. There are 17 national programs and a great number of local programs. Three out of the 17 national programs prepare students for further studies. The others are vocational, but at the same time provide general eligibility for higher studies. This provision of general eligibility is another way that the Swedish system increases access—in this case, access to higher studies.

Access to mathematics in upper secondary school is increased by the policy that makes Mathematics A—the first course in mathematics—compulsory for all students regardless of program. Upon completion of this course, students take the Compulsory National Test in
Mathematics A, then proceed through their program. The natural science program is one of the three national programs that prepare students for further studies. Students who enter this program take the most mathematics. There are seven mathematics courses available. Mathematics A, B, C, and D are compulsory. Mathematics E, Discrete Mathematics and Extension Mathematics are optional (See http://www.skolverket.se/pdf/english/natsci.pdf for details).

In each course in the degree program, students receive grades that show the extent to which pupils have achieved the objectives set down for each subject. Grades are given in three steps: Pass, Pass with distinction, and Pass with special distinction. The criteria are decided on a national level for each grade. Upon completion of upper secondary studies, students receive a leaving certificate that partially fulfills the requirements for entrance into higher education. By 2000, 73 percent of the students who entered in 1996 earned a leaving certificate (most of them after three years). In order to enter higher education students must pass at least 90 percent of subject courses in the degree program. In 2000, 80 percent of students with leaving certificates fulfilled the requirements for higher education and were eligible to enter universities. This means that approximately 57 percent of that age group (entering upper secondary in 1996) attained general eligibility for higher education before the age of 20 years. This figure demonstrates the outcome of Sweden’s policies for increasing accessibility.

**Strengths, Weaknesses and Problems**

**Strengths**

The primary strength of the Swedish approach to accessibility is that there are no insurmountable impasses in the system. The compulsory components in Compulsory Basic School and Upper Secondary School ensure access to a relatively high level of mathematics for
“all”. Alternative options for admission to Upper Secondary School increase access for students who do not fit the standard mold, and Municipal Adult Education gives everybody opportunities to study courses in mathematics at the upper secondary level (or compulsory level) and complete their education. For students who are interested in furthering their mathematics education, it is always possible to choose more mathematics courses.

The primary strength of the approach to excellence is that there are two types of standards. The aspiration objectives that frame teaching help to minimize tendencies to translate minimum expectations for competency into maximum expectations for achievement. The objectives that must be achieved send a message to teachers, students, parents, and society that all students are capable of learning at a high level. Coupled with the lack of insurmountable impasses, the Swedish approach recognizes and respects diversity in pace and circumstance and gives students opportunities to soar as high as their motivation can take them.

Weaknesses

The primary weakness of the Swedish approach to accessibility is that efforts to promote accessibility divert some resources from promoting excellence. While there are new programs aimed at enriching the experiences of gifted and strong students, there is concern that some very gifted students do not get appropriate material or challenges. Research indicates that very gifted students do not learn much during compulsory school, especially during years seven through nine. As a result, there is concern that we may lose some potential science students who choose other programs in upper secondary because they do not receive adequate stimulation in science subjects during years seven through nine. Even among those who chose to enter science, there is a great deal of concern that by the age of 18, when they leave upper secondary school, they may
have developed their ability much less than could have been possible if they experienced more challenges earlier.

The primary weakness in the Swedish approach to excellence is that despite our efforts, many students do not attain the goals. In the compulsory school 5-10 percent of students do not pass when finishing compulsory school in year nine. In upper secondary school, approximately 30 percent of students fail the Compulsory National Test for Mathematics A. These figures suggest a need to adjust our approach, especially at the upper secondary level, so that more of the students who have access to higher level mathematics also achieve a higher level of excellence.

Problems

The primary problem with the Swedish approach relates to student performance on assessments. The general tendency is that students entering the upper secondary or the tertiary level do not perform well on different diagnostic entrance tests. This is a general problem experienced at many schools and universities and may also apply to subjects other than mathematics. If 98 percent of students leaving compulsory school and 80 percent of students with leaving certificates are able to pass 90 percent of their courses, poor performance on diagnostic entrance exams suggests that there may be a problem somewhere in the system. Whether this is a problem within the curriculum, with teaching, with the assessments, or the result of a combination of these is a topic for research. As this problem is not specific to Sweden, we are interested in learning what people in other countries do about it.

Gerd Brandell
Center for Mathematical Sciences, Lund University

I have interpreted the topic for this introduction as a general issue, not related to one specific problem within mathematics education. In one of the examples I will touch upon the
question of excellence and accessibility in mathematics education, the other topic for the session. The question could be reformulated as “How are power and influence distributed among various authorities, professional groups, and other identifiable groups on the development of the school system and specifically on mathematics education?” The basis for an understanding of this matter is knowledge about the steering system and it’s functioning.

The steering of the school system in Sweden is based upon the principles of a goal-result-oriented steering process. The central steering is realized through national goals for the education. The results of the implementation in teaching and learning are evaluated and estimated through a system of national assessments and evaluations. The means to attain the goals are on the other hand in principle delegated to the local level (i.e. the local municipalities). There are several arguments for such a decentralized system. The main one is the conviction that an effective system requires that decisions be made by those who have close knowledge about the conditions in every specific case. At the same time this steering system guarantees equity on a national level through centrally defined goals and national evaluations. Hence the system ideally combines equity on a national level with local power to shape the content of schooling.

There is a general consensus among most involved – for instance all political parties – about the principles and ideal functioning of the steering system. But it is apparent that there are many obstacles for an ideal functioning of the system.

Description of the influencing bodies and parties

A comprehensive description of the steering system and other bodies that exert influence upon mathematics education would take up too much time and leave no room for problemizing and discussion. Therefore, I will build my presentation upon a general structure into which the
various “constituencies” fit. The structure is presented below. In Appendix F the authorities, bodies and lobby-groups are shortly described according to the structure. Examples of importance for mathematics education are given in the next section. In a closing section I will then discuss strengths and weaknesses of the system related to the general question about influence and power. Finally I will put forward some questions for discussion.

The three sectors are the following:

I. The political system, public authorities and society in general (i.e. industry)
II. School, teachers, teacher education
III. The academic world

The borders between the three groups are not completely clear. Teacher education is for instance closely related to the academic world in some respects, but more to the school system in others.

Within each of three sectors I distinguish between two levels:

A. National level
B. Regional and local level

The structure is tentative and does not work in all cases. The list presented in the sections to follow is not complete. The intention is to give a general idea of what kind of “constituencies” there are (See Appendix F).

Examples of processes and reforms, illustration of input and power

- Reformated teacher education

The reformed teacher education will be starting in 2001/2002 and fully implemented by 2005. A simplified and very short analysis of the process is interesting as it shows the complex pattern of influencing groups active during the process. The reformed teacher education was preceded by not less than three investigations during the years 1994 – 1995. These were initiated at different levels, one by the Parliament, one by the National Agency for Higher Education, one
by the teachers union. All investigations were initiated on the presumption that the quality of
teacher education needed to be improved, but the efforts proceeded from very different
perspectives. Some conclusions pointed in opposing directions. One main goal for the
government, the parliament, and other groups has been to integrate teacher education more fully
into the academic world and connect more closely to research in education and didactics.
Another goal is to connect closely to the profession as a teacher and give high priority to
teaching practice. These goals are not easy to adjust. There also exists a historically-rooted
discrepancy between teacher education for younger and older children that creates many
problems for the united structure desired by the government and some other groups but far from
all.

Hence, the group commissioned in 1997 by the government to work out a new teacher
education structure had a very delicate task. They presented a new structure in a 1999 report.
The suggested model was criticized by many parties during the procedure preceding the
government decision, mostly from universities and from representatives for the existing teacher
education programs. Mathematicians and mathematics educators were worried about lowering
entrance requirements. They also saw a risk for a reduction of the mathematics content within
different options of the programme and possibly also of mathematics education. Finally the
decision taken by the parliament in 2000 was very close to the original suggestion and showed
no evident revisions due to the critic from the mathematics community. However, the structure
leaves great freedom for universities to shape the programs.

One interpretation of the outcome is the following. The issue was very controversial. In
order to satisfy several parties the decision did not really settle all of the controversial questions.
The conflicts were thereby moved to the local level where the solution will depend on the relative strength of different interests within each university and among other local or regional influencing groups. On the other hand, a united structure is now adopted, which was one main political goal for the government; teaching practice has been strengthened and possibilities for universities to demand higher scientific quality of the degree paper have been created. In this sense a compromise has been reached.

The outcome of the future implementation is very difficult to predict. Several of the universities had great difficulties to negotiate among themselves and make a decision on the implementation. Some university administrations found this so difficult they even demanded an extra year for preparing their programs, a demand that was not accepted by the ministry.

- **Curriculum reform**

  A revision of curricula of mathematics at compulsory and non-compulsory school took place in 2000. It was adopted in 2000 and will be fully implemented in 2002/2003. I will not go into the whole revision but only choose a couple of questions related to the mathematics syllabus for upper secondary level. In the last big reform in 1994 the school system at upper secondary level was unified, and 17 national programs introduced. Almost all Swedish young persons (96 percent of an age cohort) attend one of the programs offered at upper secondary level – whether it is vocational or primarily preparing for further studies. There is a common core for all programs.

  One basic course in mathematics, called mathematics A, is part of the common core. Many mathematics teachers find it difficult for such a diverse group of students to take a common course at this level (at 16 years of age). The student group is diverse in different respects: in
aptitude, interest, mathematical competence, self-confidence and beliefs about mathematics. For some Mathematics A is their final mathematics course, for others it ought to create the basis for a series of other mathematics courses, i.e. in the science program. There are insufficient results among students at some programs on the national tests on mathematics A, with an extremely high failure rate. Therefore, many teachers felt a need for a revision of this course, as it was not challenging enough for students in the science program and at the same time difficult to grasp for some of the students in vocational programs.

One suggestion put forward to the National Agency for Education from teachers was to introduce two or three different courses, each designed for a group of programs. Another was to let some vocational programs leave out the mathematics A course and instead integrate mathematics into the other subjects. From several different parts of the mathematics community there was a manifest support for the idea of revising the Mathematics A course in some way. Other constituencies, such as some universities and professional organizations, also supported the idea.

The problem with the Mathematics A course touches upon the issue of excellence and accessibility. Mathematics A is required for access to higher studies. The same is the case with the rest of the common core. The current system gives anyone who has successfully completed a program (any program) at upper secondary level the right to enter some university courses. If Mathematics A was not compulsory it would be difficult to uphold this system, and students would have to supplement their skills before being admitted at university. This is a political issue of great ideological importance for the government (social democrat) and neither the
minister nor the Agency for Education were willing to listen to the suggestions about a revised course A.

Another issue was more successful for the mathematics community. In the reformed program starting in 2000 a new branch of the science program is introduced, namely a mathematics-computer science branch. It is one of three options, the others being a “science-science”-branch (emphasis on mathematics, physics, chemistry and biology) and an environmental branch (emphasis on chemistry, biology and ecology). The computer science/mathematics branch was introduced during the process due to input from the mathematics community and some universities. This illustrates the other side of the accessibility/excellence coin. The new branch will give students a new possibility to specialize in mathematics already at the upper secondary level. There was no considerable opposition towards this idea, a sign of the fact that most parties realize the growing importance of some students getting a more advanced mathematics competence at this level.

Other examples

Other examples of recent reforms and initiatives are the following. In each case it is possible to make a similar analysis of the influence of several different constituencies.

- Steps to reduce problems related to lacking mathematical competence appearing during transition from secondary to tertiary level (government decision was expected in autumn 2001, but has been postponed).

In both cases the mathematical community, especially some university departments, the ICME-SE, and the NCM have been very active promoting investigations and putting forward suggestions.
Strengths, Weaknesses, Opportunities and Obstacles

Strengths

The steering system is consistent with the principles of a goal-result-oriented steering process. The central steering of schools takes place by national goals and national assessment that guarantee equity on a national level while the means to reach those goals are in principle delegated to the local level with close knowledge about the conditions in every specific case. There is a general consensus among most involved about the principles and ideal functioning of the steering system. There is much room for creative initiatives at every level. Teachers have great responsibility and freedom.

Some schools have a special teacher appointed by the head to act as a leader of the group of mathematics teachers and to support colleagues with a competence development program. If the group of teachers work well together and have support from the head and the municipality, these schools have every chance of succeeding with their mathematics program.

Municipalities are responsible for the competence development of teachers, and there is an agreement on the extent in time every year. This agreement leaves room for competence development in mathematics and mathematics education that can also take the form of sharing ideas with teachers at other schools.

Weaknesses

There is no systematic evaluation of curriculum reforms and of current pedagogical practice. If this was established much of the input from different bodies could be handled in a positive way through such a process. There is no systematic influence from teachers or other parts of the system on textbooks and no feedback or control of the pedagogic quality and the
agreement with the curriculum and syllabuses in textbooks other than those initiated by authors and publishers. There are no national guidelines or manuals for school mathematics, which may or may not be a weakness.

A strong municipality may challenge the system and stretch the limits further than the government actually is prepared to accept. Such conflicts exist between Stockholm (with a liberal majority) and the social democrat government. (These controversies may be also viewed as a strength!).

The great majority of schools lack teachers with qualifications as researchers in mathematics or mathematics education. For upper secondary schools this should be the case according to the law, but schools do not succeed in recruiting these teachers, and many heads probably do not give priority to the problem. Mathematics is often associated with natural science in general, and programs for enhancing mathematics, science and technology are based on what scientists and engineers find useful. Very little input from university mathematicians finds its way into the system. For decades university mathematicians have kept out of the debate about school mathematics with few exceptions.

Opportunities

Most big mathematics departments now support the idea of developing graduate programs for research education in mathematics, specializing in mathematics education. Eight departments at different universities will take part in a new research education program within the national graduate school starting later this year. This marks a new sense of responsibility and readiness to get involved with school mathematics from university mathematicians.
Stockholm (the largest municipality) and several other municipalities are prepared to support teachers substantially to get training as researchers or participate in a masters program in mathematics or in mathematics education (or in other subject areas).

Subject area didactics tend to attract relatively more interest from politicians and from some important funding bodies compared to general education that has until now heavily represented research related to schools and education.

Obstacles

Since 1991 teachers in public school no longer are employed by the state but by the municipalities (or by private bodies running schools). Many teachers still deeply regret this reform and find their conditions deteriorated and their influence weakened. There is a serious lack of confidence between teachers and school politicians in many municipalities.

The informal division of the ministry into two parts – each represented by a minister - hampers reforms with common and simultaneous action in school and universities. There is also a marked discrepancy between the goals expressed by the school-side and the university-side of the ministry. The two agencies represent different cultures. The national agency for education is dominated by general educationalists with a view on skills and knowledge that differs in some respects from the demands of the universities and the view that is dominating in the national agency for higher education.

No expertise in mathematics or mathematics education is to be found on a regular basis in the ministry or the national agencies. Personnel representing such competency may be recruited to a position in any of the agencies, but in those cases, it often happens more or less by chance. Most municipalities also lack specific expertise in mathematics education on a central level.
Almost no bodies or networks exist that cover pre-school and school or school and university. This is both a local and a national problem. The barriers between the different levels are very real and have a negative influence on the system. The creation of ICMI-SE is an effort to address this lack of a common place for discussion.

Many schools do not have access to a qualified and experienced teacher who may act as a driving force at the local level. Many heads of schools are not sensitive to the need of such a teacher.

Participants identified the similarities and differences represented in Table 8.

Table 8 Sweden: Similarities and Differences Among Participating Countries with Respect to Issues of Excellence and Access

<table>
<thead>
<tr>
<th>Country</th>
<th>Striking similarities to Sweden</th>
<th>Significant differences from Sweden</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Very little attention is given to talented students</td>
<td>The average number of years in school in 1990 was 5 years. Excellence is a notion that only makes sense for upper and middle classes, for entering universities. Math courses are not differentiated.</td>
</tr>
<tr>
<td>Egypt</td>
<td>Math is compulsory up to grade 8, and must also be studied for one year in secondary. Ministry has adopted the slogan “excellence for all,” but there are doubts about whether this is relevant to reality.</td>
<td>Not all students continue their education. Nowhere near the 98% figure for Swedish students going on to secondary education. At the end of elementary school, students take an exam to enter high school classes for talented students. About 25% of total number who go to secondary school qualify for these classes, and 1/4 of classes in secondary school are reserved for excellent students. They take special courses and exams. There are few special provisions for gifted students. In general, there is no enrichment study (with exception of the special courses mentioned above). Often if a student passes</td>
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</table>
France

Preschool education not compulsory but very well developed.

Government trying to enforce the idea of taking care of everyone’s abilities in math. Two years ago support was provided to weak students in senior high school as part of regular timetable of the classroom. In French and math, one hour per week is devoted to assisting students who are having difficulties.

Lack of students for sciences (not so much for mathematics). Tension between emphasis on accessibility for all students, and excellence.

Teachers have been educated to believe that all students have the same right to be educated, so many don’t think we should make differences. The belief that each child should have the same opportunities is deeply held in French society. Have accepted the decision to make special courses for gifted students. Those done in private settings do not seem to be well accepted.

In France there is only one ministry. There used to be two, but not anymore.

India

Compulsory in secondary education up to grade 10.

In a compulsory education, gifted students do not seem to get enough challenges.

Students backgrounds range from tradition of education for generations to first generation students. Teachers have difficulty in catering to needs of all these students. Usually the teacher focuses at middle level; thus for gifted students, it is not challenging enough.

Was a movement to establish special schools—funded and run by the government—to ensure that the exam for talented students, something small might be added into their programs.

While compulsory education in India is similar (ages 6 – 14, grades 1-8), the secondary stage ends at age 16, not 18.

Sweden has exam end of grade 9. In India it is at the end of grade 10.

Sweden has different courses (A, B, C) for students at different levels. In India, the same mathematics is learned by all with no course differentiation.

Excellence is not much in the forefront. Many focus on doing well on exam. Outside agencies provide extra enrichment.
<table>
<thead>
<tr>
<th>Country</th>
<th>Description</th>
<th>System for excellent students to skip in science and math; very few use it.</th>
<th>In a large city like Tokyo, excellent students tend to go to private school, or school attached to national university, rather than public school.</th>
<th>Don’t have two ministers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>Compulsory education</td>
<td>Approximately 95% of students go to upper secondary school.</td>
<td>Very few drop out.</td>
<td>Almost all will automatically go on to next grades.</td>
</tr>
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<tr>
<td>Kenya</td>
<td>Compulsory math during the same time frame—in Kenya that means up to form four.</td>
<td>There is no course differentiation. All students follow the same syllabus.</td>
<td>Children do not go to school according to age. They are supposed to go from age 6 – but we are not surprised when an 8 year-old arrives to start. There can be age differences of up to three years in one class or grade.</td>
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<td></td>
<td>Lose very gifted students. Some lose interest, and some drop out of school altogether and cannot afford to continue in education. Have started financial aid program for students who make it up to form one. If they cannot afford the fees, the government will put them through school. Those who do not make it to that level do not benefit from this system.</td>
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<td></td>
<td>Children also have more than one opportunity to make it to secondary school. A child who doesn’t pass elementary well enough to go to secondary school might choose to repeat a year, get better marks, and go to a well-established secondary school and have better opportunities.</td>
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<tr>
<td>United States</td>
<td>Many feel that there are not enough opportunities for gifted students, but there are programs for struggling students.</td>
<td>Trying to have all children take the same course, but often courses have same names but very different math</td>
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<td></td>
<td>Do have programs such as Advanced Placement where high school students take university courses in high school, take a national exam, and receive university credit or placement depending on their scores.</td>
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Themes That Arose from the Discussion

Theme 1: Increasing access via compulsory mathematics

Mathematics is compulsory at primary level in all countries and at secondary level in many countries. Many countries have also implemented “second-chance” and, where relevant, financial support policies designed to promote access to secondary mathematics. Countries have large differences in terms of percentages of people going through the system. Some countries are considering how to reduce streaming/tracking practices that limited access to secondary mathematics. A very relevant issue seemed to be the focus of the system as a whole: What does the system steer against or aim at – passing a final examination, achieving entrance to the next level, or educating the whole population?

Illustrative quotes

“A lot of research has been done on grouping. One result is that grouping does not make any difference if the teaching, contents, materials, etc., are not improved.” (Brandell)

“Tracking is unfamiliar to Japanese society, which has an emphasis on equality. In school system there is no tracking. In classrooms at all levels, students are heterogeneously grouped.” (Hashimoto)

“In the U.S., there were earlier systems with different mathematics courses for different student groups. For instance shop classes did some mathematics that was mainly reinforcing arithmetic and not introducing new concepts. This system is now largely abandoned because it meant very small possibilities for these students to continue at the next level.” (Eddins)

“Teachers have different goals with their teaching depending on their views but also on the examination system. In some systems the teacher finds his/her most important goal to be that all students pass an exam or reach a certain level; in others to help as many as possible to pass some entrance exam; in others that some/many students reach high results.” (Brandell)
Theme 2: Challenges to increasing access

Efforts to promote access led to different outcomes in different nations. The success of policies for promoting access to secondary mathematics for all students is tied to variations in participation at the secondary level. In some countries, such as the U.S., Japan, and Sweden, participation is very high. This is not the case in countries such as Kenya, India, Egypt, and Brazil. Also, policies that tie secondary school admission to examination results present challenges to promoting access. The demographics of students who sit for exams may not reflect the demographics of the population. As a result, increases in access may not be observed among students from particular ethnic groups or socioeconomic classes. With the exception of Sweden and Japan, in nations where admission tests are used, these variations significantly influence the outcomes of efforts to promote access without reducing expectations for achievement.

In addition, rates of failure vary considerably across demographic groups. Students dropping out to work or because of repeated failure was a common concern.

Illustrative quotes

“In Kenya many talented students are lost by poverty; the family cannot afford the schooling.” (Shikuku)

“Any country wants to have excellence in any profession; similarly any country would like to have good mathematicians. At the same time, they have a concern for all.” (Agarkar)

“It used to be that students going on to secondary education would begin different kinds of study; those going to university more formal courses; students intending vocational areas, or not strong in arithmetic – might review arithmetic, apply it, in shop classes, consumer classes. This was a concern, because it meant very different opportunities for students who did not do well. Students were put into a track (streaming) that was very difficult to move from. If students began in vocational track, they couldn’t move to university track. Too often, these decisions were not necessarily made on academic grounds, and instead students from minority groups, low
socioeconomic status, etc. were put into the low tracks. But, like France, we have very strong commitment to equity and equal opportunity for all.” (Eddins)

Theme 3: Challenges to promoting excellence

Both across and within participating nations there are significant differences in approaches to defining and assessing excellence. In some of the participating countries no special effort has been made to define excellence. In others, there are ongoing debates over the meaning of excellence, how it can best be achieved, and how it is best measured.

Illustrative quotes

“Most systems are very eager to handle the accessibility problem. Few or no systems can cope with the excellence problem. Most teachers are not capable of addressing both questions inside the classroom. It is difficult to reach both the low attaining and the talented students in a classroom teaching situation involving 30 students or more.” (Shirali)

“In Brazil, it would be inappropriate to stress excellence too much when access to education is such a problem. Ten percent of the population holds 90 percent of wealth, and many students leave the system in order to work. Many are in poor health or have other problems related to poverty. It is more important to focus on addressing this than on these talent issues. Excellency is only for the middle and upper class. If we stress excellence too much, there will be higher rates of failure and even more students would drop out.” (Lins)

“.We have new attention to applications, uses of mathematics, use of situations. This is accepted by some schools, not by others. Have found that many students thought not to be talented in math had not found the mathematics being taught worthwhile. Some of them are now doing very well. Some of the students who did well when all we taught were algorithms aren’t doing as well because now they have to think more deeply. Still a turmoil –there are disagreements about this “integrated” approach. Mathematicians worry about whether there is enough preparation.” (Eddins)

Theme 4: Special provisions for struggling and talented students

In some of the participating countries, addressing the special needs of struggling and talented students is the responsibility of the family, not the schools. In these countries, although there may be some special programs or schools, most students who receive extra help or enrichment...
obtain it during private lessons from tutors. In countries where addressing these needs is primarily the responsibility of the schools, many challenges exist in identifying these students and addressing their needs. In most of these nations, exams results are used as a guide. This is relatively effective in identifying students having academic difficulty but is often less effective in identifying talented students. Some students who could perform well do not gain access to special classes, and some students who gain access do not perform as well as expected. Also, some students who gain access do not have experiences that encourage them to pursue higher education in math and science fields. These outcomes raise questions about both the identification process and the content of the classes.

The very existence of these programs has conflicted with views on equity in many of the participating countries, and the concerns raised by identification processes have exacerbated these concerns. In many of the countries, special programs for struggling students have been implemented and maintained, but many programs for talented students have been abandoned for equity reasons. There are many concerns about how to provide for students who have the potential to become scientists but emerge from the regular system unmotivated and/or unprepared to do so.

**Illustrative quotes**

“In Sweden the experience from the mathematics special class in upper secondary is that tests are not a perfect way of identifying the students. We miss some students this way.” (Gennow)

“In France, there is an experience of finding the “school-talented” by using tests, but excellence is not the same as well-achieving. Quite often when a test is given, students who can do procedures are identified, but later we learn they aren’t as creative as we expected. Talented pupils should be identified based on more input, such as interviews, than based on just an aptitude test or performance in mathematics.” (Sackur)
“Previously, the talented students were accelerated. Most recently, we have realized this didn’t solve the problem; students would take as much math as required to get into universities; two or three years.” (Eddins)

In Brazil, as well as in the U.S., there is a focus on applications and “every-day-mathematics” as a means to interest the weaker students. (Lins)

“In compulsory school, talented students may become bored and lose interest. Many are not challenged enough in grades 7 through 9.” (Gennow)

Key Questions

Key questions about access and excellence include those related to:

All students

- How do we reach all students in the process of teaching?
- When and how are weak students identified and what support do they get? What strategies can be used to engage students who have difficulty?
- What is the system attitude towards dropouts? What policies work to prevent dropouts?
- What happens in other countries to students who do not attain the goals?

Nurturing talent

- When and how do we detect talented pupils?
- What kind of support or challenges do talented students need? What do they get?
- How can we retain talented students in mathematics?

Policy and Practice

- What can be done to bridge the controversy about education among political parties and the public?
- How do conflicts among politicians, the academic world, society, schools, teacher educators, and researchers affect practice?
- What is the influence of political decisions on programs for excellence and accessibility?
Issue 7: Math Education as a Profession

What is the role of mathematics education as a profession and of mathematics education research in your country?

Brazil: Romulo Lins and Carlos Alberto Francisco

Opening Statements

Carlos Francisco
State School Secondary Joaquim Ribeiro

I will talk about my job to show the role of mathematics education as a profession. I was chosen by the school according to my classification on the tests – a math test and a pedagogical test. I worked in two schools because there was a decrease in my school, and it is hard to take part in the development of pedagogical projects in two places. Every year the team of teachers changes. All teachers take part in two or three meetings a week to discuss the educational situation of the school. Many teachers think it is boring because they fill out forms that do not work to reform the school. Teachers do not like to talk about their problems with other teachers. Salary is a problem. A teacher who has 25 lessons a week earns about $320 per month. A professor can earn up to four times more because their job is more valued. To earn more money, teachers have to take more classes, which is bad for work quality. Teachers can have up to 32 classes a week. In a high school course, teachers have three to four math lessons per week per group, and four to five per week in an elementary school. Groups have about 36 students, and teachers teach about 10 groups.

Most teachers use textbooks for their math content. These influence the practice of teachers. Mathematics education has been changed in relation to its use and function. The main
objective is to use math as an instrument to interpret reality in a critical way; reality education through math. Today education policy pressures teachers to work on interdisciplinary projects. Teachers do not, however, always know the mathematics needed for this new proposal. Teachers don’t want this kind of change. They need autonomy, good background, salaries, and engagement with teaching.

External evaluation tests are used to track student performance, but social problems have not been taken into consideration by these tests (drugs, families, etc.). Math teachers cannot teach math content because of these problems. Thinking about mathematics education and testing raises some basic questions: How must external testing happen, taking several social problems into consideration inside the schools? What must be the characteristics of these evaluations to motivate these new pedagogical practices? What will happen if we don’t adapt to this situation?

For me, educational policy minimizes social exclusion – a school is considered an excellent school if it has small rates of failure and school evasion. It does not guarantee the quality; student presence is not enough to make a student take part in society. Parents should take part in school life to improve the school.

Romulo Lins  
State University of Rio Claro

The academics have done very little to help teachers change the situation described by Carlos. Despite the fact that we are a big system, there is still small professional association. These associations should be able to effect education at the local level through the ministry or states. Brazil has 40 million students, 300,000 math teachers, and 1.4 million teachers. The
critical question is how to reach all of these. The Brazilian Mathematics Society has eleven thousand members; around two thousand are university members. There are five journals reporting research, five hundred copies every six months. The biggest impact of the society is having meetings – conferences, research seminars, meetings with courses for teachers.

Educators keep in touch with research by reading the papers directly or going to conferences. Most people working on policies – guidelines, textbooks, etc. – have an interest in research but are not researchers. They are academics serving as consultants, invited to do a task by the education group inside the ministry. “Didactical transposition” ideas come from somewhere and filter through the system: problem solving, working with patterns, investigations. Only two of 30 textbooks are clearly research informed. Publishers will not publish a book that pushes too far.

The most important part of research is building a body of socially and culturally contextualized knowledge. Francisco would like to see more research that acts together with the teacher. Main areas of research seem to be teacher education, ethno-mathematics, algebraic education, technologies, and mathematics education. There are only two pure post-graduate programs in mathematics education in Brazil. The other programs are part of education departments.

Until recently, mathematics education research existed inside the mathematics community. It was problematic to use money for mathematics to fund research in mathematics education. Now it is possible to create a separate committee on mathematics and science education. The opposition was very noisy, but a new program is being shaped that is separate from both mathematics and education. In the past, committees for book assessment, guidelines for teacher
education, and so on were dominated by mathematicians. A critical question is who should appoint these committees? Ph.D. researchers come from several communities. Some are teachers, and this number is increasing because the government is putting pressure on teachers to get post-graduate education. Many are already professors who come to improve their professional performance. Increasingly, people graduate and go straight to post-graduate studies. While teachers are undergrads, they participate in projects so that when they leave, they have an impression of the profession. Too often, they take courses without knowing why and for what purpose: There is a need to have a good association for the profession of math education, including teaching. One must have a system of scholarships that is efficient.

Perspectives

The Brazil Society of Mathematics Education is growing rapidly and has a series of books in five volumes; each sold an average of 3000 to 4000 books. Society authorities are increasingly recognizing and talking to the community, not on a one-to-one basis but as a community. As a result, they have access to committees as a representative of the community, which is a big change. What is necessary is much more public visibility including talking about research. The media says what math education should be and what is wrong and what is right with it, and there is no one who can speak out against this. There is a strong need for more development, along with research. Brazil is totally isolated from the rest of the world, and publishing material for the classroom is difficult in terms of the culture.

Participants identified the similarities and differences represented in Table 9.

Table 9 Brazil: Similarities and Differences Among the Participating Countries Related to Mathematics Education as a Profession

<table>
<thead>
<tr>
<th>Country</th>
<th>Striking similarities to Brazil</th>
<th>Significant differences from Brazil</th>
</tr>
</thead>
</table>

Mathematics Education Around the World: Bridging Policy and Practice. Reflections from the 2001 International Panel on Policy and Practice in Mathematics Education
<table>
<thead>
<tr>
<th>Country</th>
<th>Problems motivating students</th>
<th>Weak funding for research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egypt</td>
<td>Research is important field, researchers involved in teacher training,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Link between texts and research is weak,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension between mathematicians and math departments.</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>Research is important field, researchers involved in teacher training,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Link between texts and research is weak,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension between mathematicians and math departments.</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>Salary structure is low for teachers;</td>
<td>Teachers raise income by tutoring.</td>
</tr>
<tr>
<td></td>
<td>Little link between academics and classroom teachers.</td>
<td>Discipline is not a problem, no exam to be a teacher,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Research not always related to practice.</td>
</tr>
<tr>
<td>Japan</td>
<td>Role of texts</td>
<td>Well established mathematical society;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Elementary teachers have fair background in mathematics, teachers discuss teaching with each other.</td>
</tr>
<tr>
<td>Kenya</td>
<td>Some problems with students but not yet serious;</td>
<td>Research for publishing with little link to classroom.</td>
</tr>
<tr>
<td></td>
<td>Texts not based on research.</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>Tension between mathematicians and math ed community,</td>
<td>Little research in math ed; only 30 thesis since the 70s;</td>
</tr>
<tr>
<td></td>
<td>Hard for teachers to accept new ideas.</td>
<td>Situation in schools very stable with little movement of teachers.</td>
</tr>
<tr>
<td>United States</td>
<td>Little connection between research and practice</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Little research used to develop texts.</td>
<td></td>
</tr>
</tbody>
</table>

**Themes That Emerged from the Discussion**

*Theme 1: The professional lives of teachers.*

It is difficult for teachers to focus on improving mathematics education when they are faced with enormous social problems in schools and don’t have adequate support for addressing them.

Across the countries, this was an issue. Teachers’ views toward, and involvement in, mathematics education reforms, however, vary greatly within and between the countries.
Teachers can profit from collaborating with colleagues on matters of mathematics teaching, but this doesn’t happen uniformly. Cooperation between and among mathematicians, teachers, and mathematics education researchers is rare and not part of most of the cultures represented.

Illustrative quotes

“…academics help classroom teacher very little.” (Agarkar)

“Teachers need to talk with parents, but they may have problems also. Teachers need other professionals and to talk about the problems inside the school. They have to talk with everybody to get a solution.” (Francisco)

“Most mathematicians don’t care about math ed and research in math ed.” (Bodin)

Theme 2: The development of mathematics education research as a field

Mathematics education research takes place in government-funded institutes, universities, and “virtual” graduate schools. Wide disparities in the “maturity” of mathematics education research as a field were evident across the countries as well as the nature of the work (from highly theoretical, to very practical, to socially and culturally contextualized). Funding to build and support research communities was a common concern. Proximity to mathematicians and to mathematics can be both beneficial and problematic for mathematics education research. In some cases, differences of opinion emerged about the views held by mathematicians about mathematics education work.

Illustrative quotes

“Research aims should be ‘increasing our understanding of the studied phenomena with the purpose of changing them.’ The phenomena subjected to study should be connected with their wholes, we should not depend alone on ‘experimental research and linear relations’ (e.g. correlation, regression, multi-regression…), and we should pay more attention to empirical work, case studies, and prospective analyses. Further, research findings must be regarded as means to initiate discussions about these phenomena (not to be considered as the endpoint.)” (Mina)
“The main thing is that no one in France thinks secondary teachers should do research.” (Sackur)

**Theme 3: The relationship between mathematics education research and practice.**

The influence of research on textbooks, on development of syllabi, on teacher education practice, and on classroom instruction seems to be minimal, though there are a few notable examples. There is a sense that teachers could benefit from research, but in general they are remote from being involved in it. The relationship between research and various forms of practice should go in two directions; research should influence practice, but also, the questions and problems of practice should drive research. It seemed that in many cases, researchers are not always responsive to the concerns of practice.

**Illustrative quotes**

“There is a tension between what policy makers do and who they listen to.” (Eshiwani)

“There are no quick solutions from research.” (Bodin)

**Key Questions**

Key questions about the role of mathematics education as a profession and of mathematics education research included those related to:

**Structure and policy**

- Is research based in government institutes or research universities? Does this make a difference about how it is used?

- How can political issues related to research communities, for example building capacity, having access to ministry, funding, be resolved?
Connections between research and practice

- Does research really recognize the true problems of schools? Do problems of practice drive research?
- How can research results be better linked to the design of instructional materials?
- What are some examples of research in mathematics that has changed teaching practice?
- Does taking research into the classroom have any effect on methodology? Should it?
- What do we know from research about teaching large classes?

Connections among research community and other mathematics education communities

- What is needed to promote involvement of teachers in research? What could be the purpose of research done by teachers?
- What are some strategies that will smooth the tension between mathematics education researchers and mathematicians?
## Appendix A

### Mathematics Education Around the World: Bridging Policy and Practice

#### Agenda

**Thursday, July 19**

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
</tr>
</thead>
</table>
| 9:00 a.m.-10:30 a.m. | Introductions
|                  | Joan Ferrini-Mundy, Michigan State University                           |
|                  | Welcome
|                  | Herbert Clemens, University of Utah, PCMI                               |
|                  | Hyman Bass, University of Michigan, ICMI                                |
|                  | Logistics
|                  | Nancy DeMello                                                           |
|                  | Seminar overview                                                        |
|                  | Joan Ferrini-Mundy                                                       |
| 10:30 a.m.-10:45 a.m. | Break                      |
| 10:45 a.m.-11:45 a.m. | **Opening Session**<br>**From Practice to Policy**                      |
|                  | Moderator: Gail Burrill                                                  |
|                  | Deborah Loewenberg Ball, University of Michigan                         |
|                  | Hyman Bass, University of Michigan                                      |
| 11:45 a.m.-12:00 noon | Introduction to Issues Sessions                                       |
| 12:00 noon-1:00 p.m. | Lunch                      |
| 1:00 p.m.-3:00 p.m. | **Issues Session One**<br>What is the relationship of national standards and national curriculum to teaching practice in your country?<br>France: Antoine Bodin & Catherine Sackur |
| 3:00 p.m.-3:15 p.m. | Break                      |
| 3:15 p.m.-4:15 p.m. | Cross Program: Geometry of Cosmos                                      |
| 6:00 p.m.-7:00 p.m. | Cocktail Social Hour<br>North Lounge                                   |
| 7:00 p.m.-9:00 p.m. | Dinner<br>Welcome<br>E.J. (Jake) Garn<br>(Former U.S. Senator from Utah) |

**Friday, July 20**

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
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</thead>
<tbody>
<tr>
<td>8:15 a.m.-8:30 a.m.</td>
<td>Summary and Daily Orientation &lt;br&gt;Joan Ferrini-Mundy</td>
</tr>
<tr>
<td>8:30 a.m.-9:30 a.m.</td>
<td>Remarks from Invited Observers&lt;br&gt;Reflections on Issues Session One&lt;br&gt;Moderator: Gail Burrill</td>
</tr>
<tr>
<td>9:30 a.m.-10:30 p.m.</td>
<td><strong>Issue Session Two</strong></td>
</tr>
</tbody>
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Mathematics Education Around the World: Bridging Policy and Practice. Reflections from the 2001 International Panel on Policy and Practice in Mathematics Education
What is the system of teacher education in your country and how does it relate to teaching practice?

**Egypt: Fayez Mina & Khaled Farouk Etman**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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</thead>
<tbody>
<tr>
<td>10:30 a.m.-10:45 a.m.</td>
<td>Break</td>
</tr>
<tr>
<td>10:45 a.m.-11:45 a.m.</td>
<td>Issue Session Two cont.</td>
</tr>
<tr>
<td>11:45 a.m.-12:00 noon</td>
<td>Taking Stock</td>
</tr>
<tr>
<td>1:00 p.m.</td>
<td>Lunch</td>
</tr>
<tr>
<td>1:00 p.m.-3:00 p.m.</td>
<td><strong>Issues Session Three</strong> Moderator: Joan Ferrini-Mundy</td>
</tr>
</tbody>
</table>

Describe the role of algebra in the middle and secondary mathematics curriculum in your country. Similarly, how are ideas from probability and statistics currently configured in your system?

**Kenya: George Eshi Wani & Beatrice Shikuku**

<table>
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<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>3:00 p.m.-3:15 p.m.</td>
<td>Break</td>
</tr>
<tr>
<td>3:15 p.m.-4:15 p.m.</td>
<td>Cross Program Activity: Statistics and Data Group</td>
</tr>
<tr>
<td>4:15 p.m.-5:00 p.m.</td>
<td>Tea and Cookies with PCMI participants</td>
</tr>
<tr>
<td>6:00 p.m.</td>
<td>Pizza Party</td>
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</table>

**Saturday, July 21**

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<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>8:15 a.m.-8:30 a.m.</td>
<td>Summary and Daily Orientation Moderator: Joan Ferrini-Mundy</td>
</tr>
<tr>
<td>8:30 a.m.-10:30 a.m.</td>
<td><strong>Issues Session Four</strong></td>
</tr>
<tr>
<td>10:30 a.m.-10:45 a.m.</td>
<td>Break</td>
</tr>
<tr>
<td>10:45 a.m.-12:45 p.m.</td>
<td><strong>Issues Session Five</strong> Moderator: Joan Ferrini-Mundy</td>
</tr>
</tbody>
</table>

How does your country handle the balance between tradition and reform in mathematics education? What do tradition and reform mean within your mathematics education system?

**Japan: Yoshihiko Hashimoto & Miho Ueno**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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</thead>
<tbody>
<tr>
<td>12:45 p.m.-2:00 p.m.</td>
<td>Working Lunch</td>
</tr>
<tr>
<td>2:00 p.m.-6:00 p.m.</td>
<td>Remarks from Invited Observers Reflections on Issues</td>
</tr>
<tr>
<td>6:00 p.m.-7:00 p.m.</td>
<td>Outdoor concert in Deer Valley: “Riders in the Sky”</td>
</tr>
</tbody>
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**Sunday, July 22**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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</thead>
<tbody>
<tr>
<td>10:00 a.m.-12:00 noon</td>
<td>Brunch Park City Grill (tentative)</td>
</tr>
<tr>
<td>1:00 p.m.-5:00 p.m.</td>
<td>Sightseeing Excursion (Sundance Institute)</td>
</tr>
<tr>
<td>Dinner on own</td>
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</tbody>
</table>
### Monday, July 23

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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</thead>
<tbody>
<tr>
<td>8:15 a.m.-8:30 p.m.</td>
<td>Summary and Daily Orientation</td>
</tr>
</tbody>
</table>
| 8:30 a.m.-10:30 a.m. | **Issues Session Six**  
Moderator: Gail Burrill  
**How does your country and culture deal with the challenges of excellence and accessibility in mathematics education? What is the balance of power and input into the system between the various educational constituencies?**  
Sweden: Gerd Brandell & Susanne Gennow |
| 10:30 a.m.-10:45 a.m. | Break |
| 10:45 a.m.-12:00 noon | Remarks from Invited Observers and General Discussion |
| 12:00 noon-1:00 p.m. | Lunch  
**Issues Session Seven**  
Moderator: Joan Ferrini-Mundy  
**What is the role of mathematics education as a profession and of mathematics education research in your country?**  
Brazil: Romulo Lins & Carlos Alberto Francisco |
| 1:00 p.m.-3:00 p.m. | Break  
PCMI Cross Program Activity |
| 3:00 p.m.-3:15 p.m. | Break |
| 3:15 p.m.-4:15 p.m. | PCMI Tent |
| Dinner on own     |                                                                      |

### Tuesday, July 23

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:15 a.m.-8:30 a.m.</td>
<td>Summary and Daily Orientation</td>
</tr>
</tbody>
</table>
| 8:30 a.m.-9:00 a.m. | Remarks from Invited Observers  
Reflections on Issues |
| 9:00 a.m.-10:30 a.m. | Synthesis and Discussion of Issues of Common Concern  
Break |
| 10:30 a.m.-10:45 a.m. | Future Directions  
Herbert Clemens |
| 10:45 a.m.-11:30 noon | Internet communication  
Suzanne Alejandre |
| 11:30 a.m.-12:00 noon | Lunch  
PCMI Tent |
| 12:00 noon-1:00 p.m. | Break  
Preparation of Report to Institute |
| 1:00 p.m.-3:00 p.m. | Break  
Institute-wide Presentation  
Theater |
| 3:00 p.m.-3:15 p.m. | Break  
Press availability (Tentative) |
| 3:15 p.m.-4:15 p.m. | Cocktail Social Hour  
North Lounge |
| 4:15 p.m.-4:45 p.m. | Closing Dinner  
North Lounge |
| 6:00 p.m.-7:00 p.m. |  
7:00 p.m.-9:00 p.m. |
## Appendix B

**Mathematics Education Around the World: Bridging Policy and Practice**

### 2001 Participants

<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudhakar Agarkar</td>
<td>Tata Institute of Fundamental Research</td>
<td>Maharashtra, India</td>
</tr>
<tr>
<td>Deborah Ball</td>
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Appendix C: Background Materials


What is the relationship of national standards and national curriculum to reaching practice in classrooms in France?

**Monitoring Mathematics Standards and Curriculum in France**

**Two French Characteristics**

1. A centralized and most
2. Strong teacher individualism associated with a host of non-official influences.

**Mathematics Curriculum Development in the Ministry: more a continuous process than delivery of regulations.**

To encourage good teacher practices and teachers personal involvement:

- A National curriculum designed in the Ministry.
- National examinations at grade 9 and 12.
- Diagnostic National Assessments at grade 3, 6 and 10. With unique software used in all schools...Primary (grade 3), Lower secondary (grade 6), Upper secondary at grade 10.
- Incentives for innovation in teaching practices
• How to promote a constructivist teaching approach associated with a student centered teaching approach, leading to learning through student activities, without having to acknowledge a final lowering in students' proficiency?
• How to set national standards without resulting in a squash of teaching and learning activities to a collection of insignificant basic and procedural skills?
• How to keep a kind of National unity in the mathematics curriculum while letting place for openness, teacher initiative, innovative practices
• How to keep the baccalaureate face value and reliability to its present level (at least !) while preventing it to destroy the efforts done for curriculum and teacher practices improvement ?
• How to take into account the links between mathematics and other topics and facilitate integrative teaching (at least in some events !), without resulting in mathematics dissolution and final disappearance?

- How to decide the more relevant mathematics contents for general education (mathematics literacy)
- Conversely, how to identify the present mathematics topics that can be remove ?
- How to keep with high standards (and which ones) without discouraging students from learning mathematics?
L’établissement

Examens

Conseils

Orientation : affectation Bulletins/livrets scolaires
Projet d’établissement (indicateurs…)
Évaluation de L’enseignant
Le système scolaire

Concours Evaluations de programme

L'établissement

Examens

Conseils de classe
Orientation : affectation Bulletins/livrets

Evaluations standardisées
Evaluations de conformité
Evaluations de rendement

Les faits d'évaluation...

Le système scolaire

Evaluations standardisées
Evaluations de conformité
Evaluations de rendement

Evaluation/controle social
Appendix E: The Question of Depth Versus Breadth—Four Case Studies from India

Shailesh Shirali
Rishi Valley School

As a teacher, I am not involved in any way in curriculum design, so I cannot authentically comment on the reasons why syllabi have taken particular forms. However, my own leanings are clear: I value depth over breadth. I feel that an understanding of the usefulness of and inherent elegance of a concept, which leads to empowerment in problem solving, has far greater value than exposure to a large number of concepts, the intent in this case being appreciation and enrichment. (It goes without saying that these considerations have to be weighed against pedagogical factors such as mental growth patterns, what concepts are suitable at what age, and so on.)

Four case studies related to depth and breadth follow. They are all from the 11th/12th standard level, but similar case studies could be made concerning the 10th standard portions.

Group Theory

The group theory component of the syllabus is very small indeed; it barely goes beyond the definition. Examples are given of infinite and finite groups, classifying them only as abelian or non-abelian. The notion of cyclic group is not mentioned, nor that of subgroups, or isomorphism of groups. Crucially, there is little scope for showing the relevance or usefulness of the group concept or for showing its real elegance.

Until recently the topic of geometrical transformations was part of the syllabus (studies as an application of matrix algebra rather than “pure geometry”). This allowed the possibility of showing how groups occur naturally, in actual application; but the option is not available to us any longer. It is interesting to speculate on why this portion got axed. I feel that it is not a
coincidence that geometrical transformations are not part of the standard syllabus for engineering entrance examination.

Here are some typical questions from the 12th standard public examination:

1. Show that the set \{1,2,3,4\} forms a group under \( x \mod 5 \).
2. Define an operation, \( \odot \), by the rule \( a \odot b = a + b - \frac{1}{2} ab \). Show that the set of real numbers excluding two forms a group under \( \odot \).
3. Show that the inverse of an element in a group is unique.

Typically, the group theory segment takes a week to cover. Which is better—a one-week exposure to groups, going only as far as definitions and examples, or problem solving in areas already being covered (for example, optimization; challenging problems in trigonometry and coordinate geometry)?

**Boolean algebra**

Much the same comments may be made for the segment on Boolean Algebra. The syllabus merely specifies that Boolean Algebra is to be presented as an algebraic structure, with a listing of axioms and proofs for some of the basic theorems. Students are expected to know how to simplify Boolean expressions using axioms of Boolean algebra, and application to switching circuits is part of the syllabus. The portion takes roughly one week to cover.

As in the case of the group theory segment, not much depth of coverage is possible; but the situation here is certainly more encouraging—simply because many students work with computers. Many of them take Computer Science as an elective and, therefore, see Boolean algebra at play in a natural setting.
A typical examination question would be to find the Boolean functions for a given circuit, to simplify the function using the axioms, and then to construct a simplified but equivalent circuit. Students are also asked for proofs of identities; two examples are given below.

1. Prove the identity $1 + a = 1$ in Boolean algebra.
2. Prove that $a \cdot a = a$ in Boolean algebra.

**Data analysis**

I consider the data analysis segment of the Indian curriculum to be highly unsatisfactory: much more so than in the two case studies just described. A heavy emphasis is placed on computation—of mean-mode-median, standard deviation, quartiles and percentiles, index numbers, different coefficients of correlation (Pearson’s product moment coefficient $r$; Spearman’s coefficient of rank correlations, Kendall’s coefficient of rank correlation), moving averages, equations of lines of regression. The emphasis on computation is easily seen in this listing. This may be contrasted with the attention given to data interpretation.

Now there is nothing stated in the syllabus that *excludes* data interpretation. However, it must be emphasized that the Indian education scene at the high school level is very highly examination driven, and it is the style and tone of the examinations that largely determines what students actually learn. In the data analysis segment, the questions met in the examination are exclusively in the area of computation, the weight given to interpretation being nil for all practical purposes. The following questions are typical. (In the first one, the command “interpret the result” is, I’m afraid, given only lip-service.)

1. The following table gives two kinds of assessment of the work of ten students. Find the Spearman coefficient of rank correlation and interpret the result. [The table follows.]
2. The table given below shows the daily attendance in thousands at a certain exhibition over a period of weeks. Calculate seven-day moving averages and illustrate these and the original information on the same graph using the same scales. [The table follows.]
It should come as no surprise that in surveys amongst students soliciting feedback on teaching styles and how much they enjoyed the various topics, the statistics segment *invariably* ranks at the bottom. Most students perceive it as a unit where “one simply applies a formula”; as a topic having nothing to do with anything interesting or relevant and requiring no mathematical expertise.

Is an alternative possible? Yes, the data analysis segment could be made more open-ended, dealing with live topical data; say data relating to environmental problems, or to economic disparities, the emphasis being not on computation of test statistics but on interpretation.

Until about ten years ago, sampling and hypothesis testing were on the syllabus (at a fairly rudimentary level), but these components have since been deleted.

**Mean value theorem**

Included in the “higher analysis” segment are Rolle’s theorem and Lagrange’s mean value theorem. The statements are presented as facts, without proof, and the examination tests only whether one can verify the theorem in a particular setting, e.g. (these are from the examination):

1. Verify Lagrange’s mean value theorem for the function \( f(x)=\ln x \) defined over the interval \([1,2]\).
2. Verify Rolle’s theorem for the function \( f(x)=\sin x \) defined over the interval \([0,\pi]\).

It is hard to ascertain what students actually make of such theorems. They seem rather obvious, geometrically, and a typical reaction is, “Why bother to enunciate them? What could be more obvious than that a curve which goes up and comes down must at some instant have a slope of zero?” As the statements are not shown to have significant consequences, they are merely islands in a vast sea of facts.
Sample examination questions

1. Find the angle between the straight lines represented by the equation
   \( x^2 + 2(\sqrt{2})xy + y^2 + 4x + 2(\sqrt{2})y + 2 = 0 \)

2. Prove by the method of mathematical induction that \( 3^{2n+2} - 8n + 9 \) is divisible by 64 for all positive integers \( n \).

3. Using the properties of determinants, prove that \[
\begin{vmatrix}
x^2 & y^2 & z^2 \\
x^3 & y^3 & z^3 \\
xyz & xyz & xyz
\end{vmatrix}
= xyz (x-y)(y-z)(z-x)(xy + yz + zx).
\]

4. A plane meets the coordinate axes in the points P, Q, R. Given that the centroid of PQR is the point (a, b, c), show that the equation of the plane is \( bcx + cay + abz = 3abc \).

5. A variable plane passes through a fixed point \( P = (a, b, c) \) and meets the axes at the variable points A, B, C. Show that the centre of the sphere ABC traces out the surface whose equation is \( a/x + b/y + c/z = 1 \).

6. Find the largest and least possible values of \( |z| \) given that \( z \) satisfies the condition \( |z| + 1/z = 4 \).

7. Triangle ABC is right-angled and has a given area. Find the sides of the triangle for which the area of the circumscribing circle is the least.

8. Show that if \( x^y = e^{xy} \) then \( dy/dx = \ln x / (1 + \ln x)^2 \).

9. Evaluate the integral (from 0 to \( B/2 \)) \( \sin 2x \cdot \ln \tan x \, dx \).
Appendix F: Balance of Power and Inputs in the Swedish System

A general structure into which the various “constituencies” involved in the Swedish education system fit is described below, including the authorities, bodies and lobby-groups. In general, there are three sectors:

I. The political system, public authorities and society in general (i.e. industry)
II. School, teachers, teacher education
IV. The academic world

I. Political world, society in general

A. National level

Riksdagen (the Swedish Parliament) has decided upon The Education Act as well as other laws governing the school system. The Parliament also decides on curricula and syllabi for subject matters. There is a permanent committee for education within the Parliament where all political parties are represented. In this committee all propositions from the government are discussed and prepared before a decision is made in the Parliament. The government and the Ministry of Education (Utbildningsdepartementet) prepare propositions to the Swedish parliament and issue various regulations concerning the implementation of the laws.

The Ministry is organized in two sections: the Higher Education and Research section and the School section and had two ministers in July 2001. The minister of Higher Education and Research (Tomas Östros) leads the ministry and is directly responsible for higher education (undergraduate and graduate) and research. The ministry has an on-going dialogue with all institutions offering higher education. A special minister of School (Ingegärd Wernersson) is
responsible for childcare (pre-school), primary and secondary education. The ministry has an on-going dialogue with the confederation of municipalities and with individual municipalities.

The Parliament or the government commission various groups of politicians or experts to investigate specific areas and propose means to provide solutions to current problems or reform part of the system. The experts may belong to sectors II or III.

Examples:

- Expert group for a reformed structure and content of the upper secondary school (the gymnasium) commissioned by the government in 2001.

The National Agency for Higher Education (Högskoleverket) is the central authority responsible for monitoring and evaluating higher education, research, and research education as well as for investigating specific topics when commissioned by the government. It is linked to the corresponding section of the Ministry. Various expert and reference groups also contribute. Sometimes some or all members belong to sectors II and III.

Examples:

- Reference group for mathematics in higher (i.e., tertiary) education during 1997 – 1999.

The National Agency for Education (Skolverket) is the central authority responsible for monitoring and evaluating all schools. It is linked to the corresponding section of the ministry. The Agency is responsible for providing national tests and guidelines for assessing these.

Examples:

- Current projects involving mathematics: “Desire to learn” and “Basic skills”.

Mathematics Education Around the World: Bridging Policy and Practice. Reflections from the 2001 International Panel on Policy and Practice in Mathematics Education
Various experts, project groups and reference groups of teachers and representatives of different communities attached to the National Agency for Education for revision of syllabuses and for carrying through projects.

Confederation of Swedish Enterprise (Svenskt Näringsliv)
Professional Organization for Engineers (Civilingenjörsförbundet)

B. Regional and local level

The Regional offices of the National Agency for Education act with great independence. The National Agency for Education is currently (in 2001) implementing a new and decentralized organization. Municipalities (289 in all) have a great freedom to shape their local educational policy. Each municipality must appoint a political committee for public education. The committee works out the school plan agreed by the municipality. The school plan specifies the measures to be taken in order to achieve the national targets.

II School, teachers and teacher education

A. National level

Universities deliver education and training for future teachers within specific teacher training programs and within more general degree programs. Starting in 2001 all education and training for future teachers will be offered within one comprehensive program with a great number of options as for levels, subjects and so on. Universities may and sometimes do co-operate when designing programs in order to get better co-ordination.

National Centre for Mathematics Education (NCM) at Gothenburg University has a general time-limited commission from the government to provide mathematics teaching and learning resources for schools and teachers. NCM investigates and suggests a program for competency
Mathematics Education Around the World: Bridging Policy and Practice. Reflections from the 2001 International Panel on Policy and Practice in Mathematics Education

development of mathematics teachers. This is a special commission from the government given in 2000. The report will be delivered to the government in 2001. Closely attached to the National Centre for Mathematics Education is the editorial board of Nämnaren, a journal targeting mathematics teachers, especially at primary and lower secondary level. Every second year the Biennalen-movement sponsors a three-day-long conference for mathematics teachers, which gathers several thousand participants who take part in hundreds of lectures and workshops. Various experts, project groups and reference groups of teachers and representatives of different communities are attached to the National Agency for Education for revision of syllabuses and for carrying through projects. The Association for Teachers in Mathematics (SMaL) organizes mostly teachers at primary and secondary level. The Association for teachers of Mathematics, Science and Technology (LMNT) organizes mostly teachers at upper secondary level. There is a network for mathematics teacher educators, LUMA-network. Finally, authors of mathematics textbooks for use in school also influence what is taught.

B. Regional and local level

Each school has to develop and agree on a plan for education, teaching and learning in which the curriculum and other steering documents are interpreted in the local environment. Mathematics teachers at every school are responsible for the mathematics education plan and for choosing educational material and methods, all within the frame of the national curriculum and the national mathematics syllabus. The freedom at the local level to interpret the curriculum and to make concrete the syllabus is considerable. Each municipality and each school has to prepare a written quality assessment report every year with the achievements relative the national goals and steps required where the objectives have not been reached. Some schools have one teacher
responsible for mathematics who is assigned by the head to co-ordinate the planning, support colleagues, and work out competence development plans. Every teacher is responsible for the teaching, assessing and grading of his or her students. The assessment is based on the national assessment program and on locally decided assessment. Also, local or regional sections of mathematics teachers associations are very active in some regions.

Each university with the right to offer teacher education and do the examining of teachers has the freedom and responsibility to define and create the goals and content of the teacher education within the overall goals and a general structure decided by the parliament and government. Until now the structure and content of different teacher education programs have been specified in some detail in a national plan set down by the Parliament. The new teacher education adopted in 2001 leaves more freedom to the universities.

Each university and each faculty offering teacher education must have a specific committee for teacher education. This is an exception from the otherwise stated competence of each university board to organize the committees for research and education at each faculty according to the local needs.

### III The academic world

Royal Swedish Academy of Science (KVA) has set up a permanent national committee for mathematics. There is also a Swedish ICMI-committee, (ICMI-SE) which is linked to KVA. The committee incorporates representatives of school and teacher education as well as higher education and research. Other groups are the Academy of Engineering Sciences (IVA), The Swedish Society for Mathematics Education Research, SMDF, the PRIM-group within the
University of Education (Lärarhögskolan) at Stockholm, which is a research and development group for mathematics assessment. The group is commissioned by the National Agency for School to provide national tests in mathematics for compulsory school and the course A at upper secondary level. Edmeas (Educational Measurements), Universities, and the Swedish Mathematical Society (SMS) constitute the remaining constituencies.

Edmeas (Educational Measurements) is situated within Umeå University. This group is commissioned to provide national tests in mathematics for upper secondary level except the course A. Both PRIM and Edmeas cooperate intensively with groups of experienced teachers for suggesting problems and evaluating the tests. Universities, besides offering teacher education, also represent the receivers of students from upper secondary schools: future mathematicians, engineers, economists and other professionals in all areas. The Swedish Mathematical Society (SMS) organizes professional mathematicians and others. A number of upper secondary mathematics teachers are also members of the society.