IAS/Park City Mathematics Institute

Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540

To all registered PCMI participants: This document contains your program's course descriptions. Be sure to read your program's section before you arrive at PCMI.

We also recommend that you take a look at the information about the other PCMI programs; most of the courses and seminars are open to all participants and you may wish to attend other sessions during your time at PCMI. Remember, interaction is a key component of the PCMI Summer Session Programming.

Graduate Summer School

Organizers: **Alexei Borodin, MIT; Ivan Corwin, Columbia University; and Alice Guionnet, CNRS-ENS Lyon**

The Graduate Summer School offers an intense introduction to problems and techniques in an active field of research. We expect about 80 graduate students to attend the Summer Session this year. This will be an exciting – and unique – three weeks!

As you can see from the summaries, the courses cover a lot of ground quickly. Many post-docs and senior mathematicians will attend the lectures alongside the GSS participants. Please feel free to ask questions to make sure the lecturers are proceeding at an appropriate pace.

There are other features of the Graduate Summer School designed to help you get the most out of the courses. The lecturers and their teaching assistants will run daily problem sessions related to the course material and will be available to answer questions. The lecturers will produce notes for lectures, and the course assistants will distribute these notes during the Summer Session.

Opportunities for mentor-relationships frequently develop during PCMI, and you are encouraged to take the initiative in establishing these relationships. There will be plenty of time to discuss mathematics in the relaxed atmosphere of PCMI, away from academic schedules and every day routines. You are also encouraged to form study groups with other PCMI students. We think you will find meeting and working with other students to be an important part of the program.

There are three to four Graduate Summer School lectures each day that PCMI is in session – except for June 28 and July 12 when PCMI takes the afternoon off.

Many of you will be tempted to attend the seminars of the Research Program as well. While we encourage you to sample them, we caution you not to burn yourselves out! Three lectures or more a day is more than most people can absorb, so please don't try to do everything on the schedule.

Graduate Summer School Course Descriptions

Each lecturer gives four lectures.

Week 1: June 26–30

Monday: Dumitriu, Serfaty, Shlyakhtenko, Fyodorov
Tuesday: Dumitriu, Serfaty, Shlyakhtenko
Wednesday: Serfaty, Shlyakhtenko
Thursday: Serfaty, Fyodorov, Dumitriu
Friday: Dumitriu, Fyodorov (2), Shlyakhtenko

Ioana Dumitriu, University of Washington
Eigenvalues, graphs, and Catalan numbers: the semicircle law and beyond

The most basic, and hence arguably most famous result in random matrix theory is the (Wigner) semicircle law. Simply stated, for any Wigner symmetric/Hermitian random matrix with independent, identically distributed entries of mean 0 and variance 1, the distribution of a randomly-chosen eigenvalue converges to the (normalized) semicircle. This result is true regardless of the entry distribution (with only very minor conditions). The proof involves linear algebra and combinatorics: in particular, looking at traces of powers of the matrix and counting walks on the complete graph to obtain Catalan numbers. We will prove this result and then delve into why much more can actually be shown. From the fact that the shape of the histogram for all eigenvalues is semicircular, to investigating the fluctuations from the semicircle, we will explore the surprisingly stable structures that emerge from the spectrum of large random matrices.

Yan Fyodorov, King's College London
Counting equilibria in complex systems via random matrices

How many equilibria will a large complex system, modeled by N randomly coupled autonomous nonlinear differential equations typically have? How many of those equilibria are stable, that is are local attractors of the nearby trajectories? These questions arise in many applications and can be partly answered by employing the methods of Random Matrix Theory. The lectures will outline these recent developments.

Sylvia Serfaty, New York University
Microscopic description of Log and Coulomb gases

As observed by Dyson and Wigner, instances of classical random matrix ensembles (such as the Gaussian Unitary Ensemble, Gaussian Orthogonal Coulomb Gases, Ginibre Ensemble) can also be viewed as systems of particles in the plane or on the real line with logarithmic or Coulomb interactions, at particular temperatures, which are also called beta-ensembles. These have been in recent years intensely studied for their own sake. We will examine general Coulomb and Log gases (including in higher dimension than 2), taking a point of view based on the detailed expansion of the interaction energy. This allows us to describe the macroscopic and microscopic behavior of the systems. In particular we will show a Large Deviations Principle for the empirical field and a Central Limit Theorem for fluctuations down to the mesoscopic scales. This allows us to observe the effect of the temperature as it gets very large or very small, and to connect with crystallization questions, such as the occurence of the triangular Abrikosov lattice. The main results are joint with Thomas Leble and also based on previous works with Etienne Sandier, Nicolas Rougerie and Mircea Petrache.

Dimitri Shlyakhtenko, University of California, Los Angeles
Random matrices and free probability

Voiculescu invented his free probability theory to approach problems in von Neumann algebras. A key feature of his theory is the treatment of free independence — based on the notion of free products, such as free products of groups — as a surprisingly close parallel to classical independence. Rather unexpectedly it turned out that there are deep connections between his theory and the theory of random matrices: very roughly, free probability describes certain aspects of asymptotic behavior of random matrix models. In this course, we will start with an introduction to free probability theory, discuss connections with random matrix theory, and finally describe some applications of results from random matrices in operator algebras and vice versa.

Week 2: July 3–7

Monday: Erdös, Tao (2)
Tuesday: Fourth of July holiday/ PCMI not in session
Wednesday: Deift, Erdös, Tao
Thursday: Deift, Erdös, Tao
Friday: Deift (2), Erdös

Percy Deift, New York University
Riemann-Hilbert problems

1. Basic theory of RHPs
2. Use of RHPs in inverse scattering theory
3. Application of the nonlinear steepest-descent method to compute the long-time behavior of integrable systems
4. Application of the nonlinear steepest-descent method to problems in random matrix theory and the theory of orthogonal polynomials

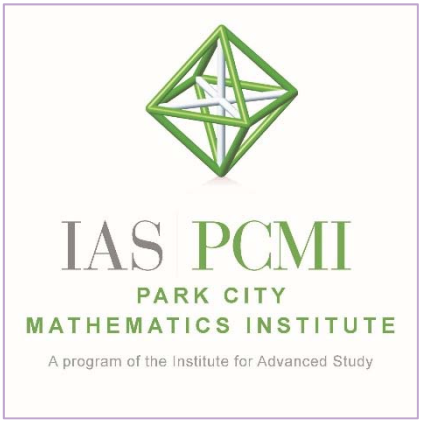
Laszlo Erdös, Institute of Science and Technology Austria
The matrix Dyson equation for random matrices

In recent years, Wigner matrices have been studied in increasing generality by gradually relaxing the original conditions that required independent, identically distributed entries. We analyze the key equation, the so-called matrix Dyson equation, that governs the density of states and the behavior of resolvent matrix elements of the corresponding ensemble. As an application, we present local laws and local spectral universality for random matrices with correlated entries.

Terence Tao, University of California, Los Angeles
Universality for random matrix ensembles of Wigner type

The lecture series will present some of the advances in the last few years (by Erdös, Schlein, Yau, Yin, Vu, myself, and others) in demonstrating universal limiting asymptotics at microscopic scales for the spectrum of random matrices of Wigner type, focusing particularly on the Lindeberg exchange strategy and the local relaxation flow method of Erdös, Schlein, Yau, and Yin.

Week 3: July 10–14



Monday: Rudelson, Quastel, Virag
Tuesday: Rudelson, Quastel, Virag
Wednesday: Quastel, Virag
Thursday: Rudelson, Quastel, Virag
Friday: Rudelson

Jeremy Quastel, University of Toronto
Introduction to KPZ

The Kardar-Parisi-Zhang equation is a canonical model for one dimensional random growth. It is also a member of a large universality class containing also directed polymer free energies, and randomly forced fluids in one dimension, characterized by nonstandard fluctuations, many of which were first discovered in random matrix theory. We will introduce the main ideas behind the meaning of the equation and obtaining exact asymptotic distributions.

Mark Rudelson, University of Michigan
Delocalization of the eigenvectors of random matrices

Consider a random matrix with i.i.d. normal entries. Since its distribution is invariant under rotations, any normalized eigenvector is uniformly distributed over the unit sphere. For a general distribution of the entries, this is no longer true. Yet, if the size of the matrix is large, the eigenvectors are distributed approximately uniformly. This property, called delocalization, can quantified in various senses. In these lectures, we will discuss recent results on delocalization for general random matrices.

Balint Virag, University of Toronto
Operator limits of random matrices

The behavior of random matrices can be understood by studying their limits as the dimension tends to infinity. The limiting operators can be used to study eigenvalue statistics. This mini-course will be an elementary introduction to this theory.

Among others, we will give a simple description of the Tracy-Widom distribution and derive some basic properties. We will also give a non-computational proof of the Wigner semicircle law. We will discuss several open problems. No previous experience with operators is required.

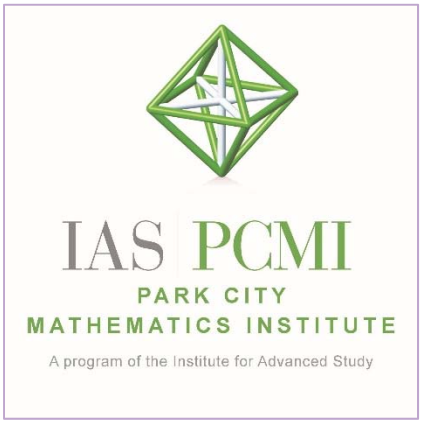
Clay Scholars

Clay Scholars have been appointed for their ability to play a leading role in the research and dissemination activities of PCMI – through lectures, formal and informal seminars, and the many informal contacts that play an important part in mathematical research. The Clay Scholars will interact with participants in all PCMI programs.

Craig Tracy, University of California, Davis

Horng-Tzer Yau, Harvard University

Research Program



Organizers: **Alexei Borodin, MIT; Ivan Corwin, Columbia University; and Alice Guionnet, CNRS-ENS Lyon**

There will be two or three research seminars on most days, depending on the preferences of the organizers and participants. Some lectures will be accessible to advanced graduate students and postdocs while others will be intended for more specialized groups of the Research Program.

Schedule: The daily seminar time slots are 9:40am, 1:00pm, and 4:30pm The majority of the schedule will be established by the organizers as the Summer Session progresses. Topics for additional informal sessions or working groups may be arranged as well.

A primary goal of the research program is to foster collaboration among a diverse group of participants. The program is designed to provide an opportunity for researchers to meet and talk together, interact with students entering the field, and hopefully even get some work done. It is expected that researchers will be active in many aspects of the PCMI Summer Session by interacting with participants in the other PCMI programs and/or attending their sessions. The rest of the time will be free for work and informal discussions.

You are encouraged to seek out students whom you can mentor. Your responsibility towards those students can be as little as "lunch once a week" or as much as you and the student want. While this is a great opportunity to meet and interact with graduate students and other research mathematicians, the same is also true with regard to undergraduate students. In past years, researchers who wished to work with undergraduates have been very welcome. Contact Tom Garrity or Irena Swanson, the Undergraduate Summer School Organizers.

The mathematicians who designed this integrated summer institute, and obtained the funding for it, have a serious commitment to working with school teachers and learning from them. You are invited and encouraged to take part. Whether you have a particular interest in education or simply want to expand your horizons, the PCMI Summer Session offers an easy and interesting opportunity to interact with school teachers and learn what mathematicians and mathematics educators are doing in the arena of education reform. For example, it is possible to arrange to attend some of the sessions in which teachers present their materials.

One popular feature of the Summer Session has been the Cross Program Activities. These activities are designed to introduce basic topics in, and explore aspects of, the area of concentration of the Summer Session and to provide a forum for interaction and discussion among participants from different programs.

We also suggest sharing lunch tables or organizing joint recreation activities with participants from other programs.

Undergraduate Summer School

Organizers: **Thomas Garrity, Williams College**; and **Irena Swanson, Reed College**

There will be 42 undergraduate students attending Summer Session 2017.

Antonio Auffinger, Northwestern University
Random matrices beyond random matrix theory

This course will focus on surprising and quite inspiring appearances of random matrices in other fields of mathematics. These include combinatorics, number theory, probability theory, statistical physics and beyond. We will discuss problems related to longest increasing subsequences, random tilings, connections to the Riemann zeta-function and spin glasses, among others.

We will give an introduction on each of these topics (no previous knowledge required!), focusing on questions where the distribution of eigenvalues plays (or is conjectured to play) a big role. Homework assignments will be a major part of the course.

Mihai Stoiciu, Williams College
Introduction to Random Matrix Theory

Initiated by research in multivariate statistics (Wishart, 1928) and nuclear physics (Wigner, 1955), the study of random matrices is nowadays an active and exciting area of mathematics, with numerous applications to theoretical physics, number theory, functional analysis, optimal control, and finance. Random Matrix Theory provides understanding of various properties (most notably, statistics of eigenvalues) of matrices with random coefficients.

This course will provide an introduction to the basic theory of random matrices, starting with a quick review of Linear Algebra and Probability Theory. We will continue with the study of Wigner matrices and prove the celebrated Wigner's Semicircle Law. After this, we will turn our attention to Gaussian ensembles and investigate the Gaussian Orthogonal Ensemble (GOE) and the Gaussian Unitary Ensemble (GUE). In particular, we will derive the joint distribution of eigenvalues for GOE and GUE and discuss the spacing distributions of the spectrum for these ensembles. The last lectures of the course will be dedicated to random Schrodinger operators and their spectral properties (in particular, the phenomenon called Anderson localization).

Undergraduates are also welcome to attend more introductory lectures of the Graduate Summer School and are encouraged to attend the Cross Program Activities. However, please don't overdo it! It is too easy to become overwhelmed by the vast amount of mathematics available at PCMI. Concentrate on what is the most valuable to you at your current stage of preparation.

USS Participation Policy: Undergraduate Summer School participants who are funded by the IAS/Park City Mathematics Institute are expected to participate from the beginning of the summer session to the end on the afternoon of July 14.

Undergraduate Faculty Program

Organizers: **Thomas Garrity, Williams College**; and **Irena Swanson, Reed College**

Victor Moll, Tulane University

There is a richness and a beauty in the study of special functions that is appealing to almost all mathematicians (despite the fact that most mathematicians only have a hazy view as to what special functions are). With Experimental Mathematics and the use of modern software such as Mathematica, this richness can be shared with undergraduates. Various special functions, under the general name 'hypergeometric functions', and their special values appear throughout mathematics, in areas ranging from number theory to combinatorics to physics. Most of the elementary functions of Calculus appear in this form.

The Undergraduate Faculty Program will explore properties of these functions using Mathematica. The data collected from these experiments will be used to generate conjectures about arithmetic properties of the coefficients of these special functions. Given that these functions have been around for a long time, it is surprising that these type of questions have not been decided. Many of these lead to beautiful pictures that await for an explanation.

The participants will be given a variety of projects at the beginning of the program. These will be flexible to match personal interests. By the end of the three weeks, ideally people will have the questions and the tools needed to work on research in special functions, both on their own and with students.

No particular knowledge of special functions is required, as the background material will be covered during the program.

Workshop on Increasing Minority Participation in Undergraduate Mathematics

Organizers: **Leona Harris, Marymount University**, and **William Velez, University of Arizona**

June 25 – July 1, 2017

Workshop Leaders: **Erica Walker, Columbia University**, and **William Vélez, University of Arizona**

Mathematical training is more important today than ever before and mathematics departments should be playing a central role in increasing the mathematical content of students' undergraduate curriculum. The demographics of the country are changing and it is important to reach out to traditionally underrepresented groups in the mathematical sciences.

One of the aims of this workshop is to explore issues related to the experiences and participation of women and minority students in the mathematical enterprise. The other aim is to provide faculty with tools to increase the number of mathematics majors and to increase diversity in undergraduate mathematics.

Participants will benefit from seeing these activities played out in real time via their constructive engagement with students and faculty in the rich environment of the PCMI Summer Session. To that end, you are also encouraged to attend the popular Cross Program Activities that are scheduled during the week.

Teacher Leadership Program

Organizers: **Gail Burrill, Michigan State University; James King, University of Washington**; and **Monica Tienda, Oak Park School District (MI)**

The Teacher Leadership Program regards the teacher as the primary agent for promoting and implementing effective mathematics teaching and learning. This year's program will have about 60 participants. The TLP is designed to enable teachers to make informed decisions and to implement change about teaching mathematics in ways that promote learning for all students.

The TLP is structured around three goals:

All teachers should be involved in

- continuing to learn and do mathematics
- analyzing and refining classroom practice
- becoming resources to colleagues and the profession.

Each of these goals is reflected in the three strands that comprise the summer courses and activities.

Goal 1 Math Course: Random Matrices

The contexts and methods developed in the course will be low-threshold, high-ceiling, with plenty of interesting mathematics for any grade level.

The course is about the nature of randomness. How do we decide something is random? How can we decide something isn't random? There are many possible distributions of random numbers and many possible distributions of random matrices. The course will investigate the ways that data can be tested to see if it comes from a random source and the ways to use random numbers and matrices to simulate real phenomena. An example:

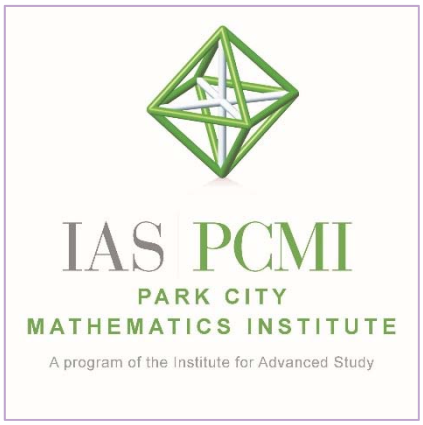
Jennie is an excellent fast-pitch softball pitcher. She has three pitches: a fastball, a changeup, and a drop curve. Jennie mixes up her pitches to try to fool hitters. Here's a sample of 24 pitches she threw during a game, in the order she threw them:
F, F, C, D, F, F, F, D, C, F, F, C,
D, F, C, F, D, F, F, D, C, F, D, F
Jennie just threw a fastball. About how likely do you think it is that the next pitch will be a fastball? a changeup? a drop curve?

The course will investigate questions like these, tying them to the precollege mathematics curriculum.

Goal 2 Reflecting on Practice: Making Inquiry a Core Teaching Practice

What role does inquiry play in developing students' insights about and understanding of mathematics? Participants will consider research related to teaching and learning mathematics with a particular focus on how active learning can produce both conceptual knowledge and procedural fluency. The discussion will be grounded in the study of lessons and classroom practice in both the United States and other countries as participants work collaboratively to better understand how they can design experiences for students that enable them to make sense of mathematics.

Goal 3 Working Groups



As part of their summer activities, each participant selected for the Teacher Leadership Program will be assigned to one of three options: 1) a small team that will focus on developing an activity related to providing resources to support teachers as they implement standards for mathematical proficiency in their schools and districts or on a TLP academic year outreach project; 2) a special working group attending a course related to the PCMI research math topic Random Matrices; or 3) a working group on lesson study, which will design and deliver a lesson to area high school students during the three-week session and revise the lesson for use by other mathematics educators. These teams have been assigned with respect to interest and experience.

2017 Working Groups:

- Supporting Teachers as Leaders
  - Algebra
  - Advanced Algebra
  - Advanced Mathematics
  - Statistics
  - Number and Operation
  - Geometry
  - Calculus
  - Social Justice in Mathematics
  - Outreach
- Lesson Study
- Research Course-Random Matrices

A LETTER FROM THE TLP ORGANIZERS:

The summer session of the IAS/Park City Mathematics Institute is fast approaching. We're looking forward to meeting you and to a program filled with intense and rewarding intellectual and social experiences. While some of you will be returning to Park City, many will be attending for the first time.

All TLP participants attend the Math Course and the Reflecting on Practice course beginning at 8:15 each morning. The afternoons will be devoted to the various Working Groups. The middle of the afternoon is typically taken up with a Cross Program Activity involving everyone at PCMI or a focus group to be defined after participants arrive. Because of the late start this year, additional sessions are scheduled on Saturday-July 1, Sunday-July 2, and Saturday-July 15. No sessions or activities are scheduled on Tuesday-July 4, the weekend of July 8-9, the afternoon of Wednesday-July 12, and Friday evenings, leaving them free for participants to use as they wish—there will be plenty of opportunity to find new friends to enjoy local events and recreational activities with. There will be optional late afternoon and evening sessions on Mondays, Tuesdays, Thursdays, and Fridays—these are scheduled to take advantage of the rich source of expertise available from PCMI, and will feature a variety of activities including group presentations, joint meetings with other groups, technology training, or sessions where participants can share activities with other teachers.

You have received a separate email assigning you to a working group. Your group may do some work ahead of time so that you will be able to know what supplies to bring.

On several evenings we will work with hands-on activities that allow us to share and get acquainted with each other. The leadership team will schedule a variety of other sessions depending on the interests of the participants. If you have interesting activities that would enrich these evenings please bring them along. If there are special supply needs, let Monica Tienda know before you come (mtienda69@gmail.com).

Some basic supplies to bring:

- Math tools: compass, ruler and calculator (TI NSPIRE CAs will be available)
- Earphones
- Laptop
- Pencils and notebooks

Do you have any "gem" problems of the day/week, that are accessible to high school students, but interesting enough to challenge teachers and mathematicians? Do you have an idea for a collaborative project that could involve participants from other programs? Do you have an idea that you would really like to share during a hands-on evening meeting? Even though we may have limited time to share these with the whole group, you will have plenty of informal time to share with your colleagues.

Our first Math Course session will take place on arrival day, Thursday, June 29, beginning at 3:00pm. This will be followed by a casual supper with participants from all of the PCMI programs at 6:00pm in the Dining Tent. The opportunity to participate in organizing PCMI's parade entry for Park City's unique July 4th celebration is scheduled for 8:00pm following the casual supper, also in the Dining Tent.

TLP Participant Policy: Teacher Leadership Program participants who are funded by PCMI and/or intending to obtain credit for the courses are expected to participate in all three components of the Program (Math Course, Reflecting on Practice, and Working Groups) from the start of the TLP on Thursday afternoon, June 29, to the end on the afternoon of July 15.

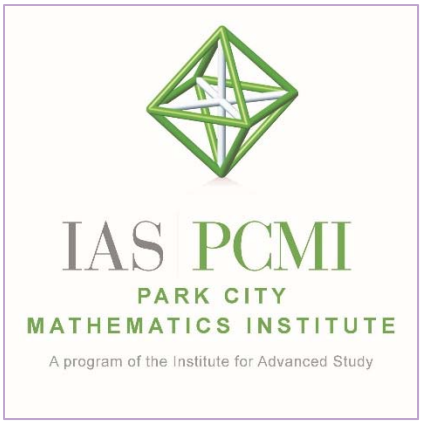
High School Mathematics Camp

Organizer: Troy Jones, Alpine School District

July 10 – 14, 2017

Twenty secondary-level students from Utah will work together each day in a problem solving mathematics course that closely follows the course given in the Teacher Leadership Program. This course is open for observation only by express arrangement with Troy Jones. He can be found in the TLP classroom at the computer each morning or in the PCMI office in the afternoon.

International Seminar on Mathematics Education



Organizers: **Gail Burrill, Michigan State University**; and **Patrick Scott, University of New Mexico**

July 2 – 9, 2017

"Mathematics Education Around the World: Bridging Policy and Practice – A Focus on Mathematics and Teacher Participation"

The International Seminar at PCMI brings together a group of mathematicians, educators, and secondary school classroom teachers from around world to establish an ongoing dialogue that examines, in practical and grounded terms, the interplay of policy and practice in diverse systems of primary and secondary mathematics education. Since 2001, the seminar has convened a small group of international participants, selected for their key roles in policy and practice in mathematics education in their own countries. The 2017 International Seminar will have participants from the Czech Republic, Guatemala, Nigeria, the Philippines, Spain, and the United States. The participants will engage in a dialogue on the preparation and development of mathematics teachers in the area of probability and its role in mathematics and the secondary mathematics curriculum.

In this context the specific goals for this summer are to:

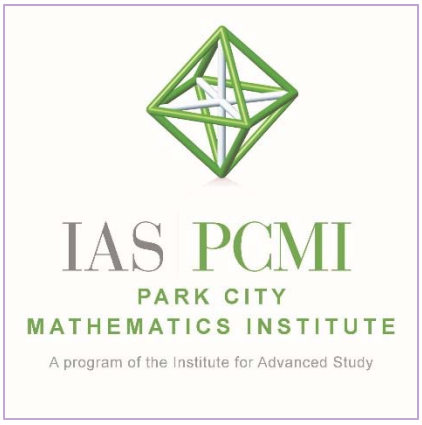
(1) Consider issues related to the content knowledge teachers need to teach probability in diverse settings within and across countries and the implications for teacher preparation and development;

(2) Develop a structure and context for sharing our work and addressing challenges through an international dialogue; and

(3) Prepare brief papers on issues related to teaching and learning of probability that will be made available for sharing around the world. To help you prepare for the seminar, some background readings, guiding questions that you will be asked to respond to in terms of your own country's policies and practices, and a draft agenda will be sent to you should you accept the invitation to participate.

Participants have already received from the organizers some background readings and guiding discussion questions that you will be asked to respond to in terms of your own country's policies and practices. A final agenda and complete activity schedule will be provided to you in your registration packet

Frequently Asked Questions (FAQ):



1. **Q**. *May I attend other seminars and courses besides my own?*
   A. Yes, if they catch your interest. Exception: the High School Math Camp (see page 10).
2. **Q.** *Are there office hours with the TAs or professors?*
   A. The Graduate Summer School TAs will hold scheduled problem sessions daily. In addition, the Lecturers and TAs have use of a meeting area for meetings with students by appointment.
3. **Q.** *Why are Cross Program Activities important to me?*
   A. PCMI was founded on the premise that everyone in the mathematics community benefits from the interaction of the various constituencies. We endeavor to schedule interesting Cross Program Activities that will appeal to most participants. And we also encourage personal interaction through occasional assigned lunch seating and through other activities such as the Parade and Pizza and Problem Solving. An added bonus are the activities organized by other PCMI participants, in which anyone who shares the same interest may join.
4. **Q.** *Is it possible to become overloaded in three weeks at PCMI?*
   A. In a word: yes. That's why July 4, the afternoons of June 28 and July 12, and unscheduled weekends are important for rest and relaxation. It is another reason to take part in Cross Program Activities and recreational events organized by PCMI, or to join with other participants' activities in your free time.

Acknowledgments

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